

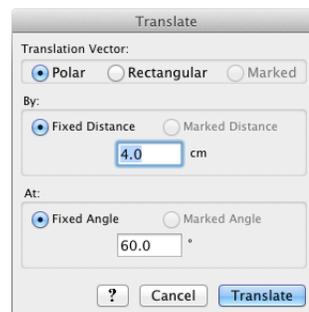
## Translation Challenges (long form) Name: \_\_\_\_\_

In this activity you will translate a point and compare its motion to the motion of its translated image.

### TRANSLATE A POINT

Begin by translating a point and describing how the translation behaves.

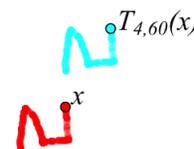
1. In a new sketch, construct an independent point  and label it  $x$  .
2. Drag the point  around in your sketch. Because you can drag this point anywhere in your sketch, and it doesn't depend on other objects, we say that point  $x$  is the *independent variable*.
3. Select point  $x$   and choose **Transform | Translate**. The *Polar* Translation Vector should be chosen.
4. Fill in the edit boxes to translate by 4 cm at  $60^\circ$ . As you do so, notice how the preview shows the position of the translated point.



5. Press Translate to make the translation permanent.
6. Label the translated point   $T_{4,60}(x)$ . (This is the *dependent variable*.)

You can read  $T_{4,60}(x)$  as “the translation of  $x$  by 4 cm at 60 degrees.”

7. Change the color of the dependent variable  $T_{4,60}(x)$  by selecting it  and choosing **Display | Color**.
- Q1** Drag  $x$  up. Which way does  $T_{4,60}(x)$  go? How fast does it go relative to point  $x$ ? Drag  $x$  to the left. Which way does  $T_{4,60}(x)$  go? How fast does it go?
8. Turn on tracing by selecting both points and choosing **Display | Trace Points**.
9. Click in empty space to deselect  both points. Then drag the independent variable  $x$  to trace out an interesting shape.
- Q2** Describe the traced shapes. How are they similar, and how are they different? Consider position, size, angle, and anything else you think of. On your paper include a drawing of your two traced shapes.
- Q3** Drag again to search for fixed points. Can you find any? Explain why or why not.
- Q4** Click the **Information** tool  on point  $x$ , and then on  $T_{4,60}(x)$ . How do the resulting balloons describe each point?

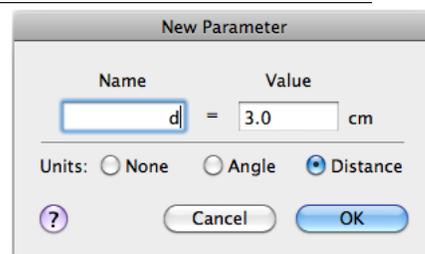


### USE AN ADJUSTABLE TRANSLATION

10. Choose **File | Document Options**. In the dialog box, choose **Add Page | Blank Page** and click **OK**.

## Translation Challenges (long form, continued)

11. Create a parameter by choosing **Number | New Parameter**. Make it a distance parameter, set its value to 3.0 cm, and name it  $d$ .



12. Create another parameter. Make this one an angle parameter, set its value to  $45^\circ$ , and name it  $\{theta\}$ .

When you type the closing bracket, the name changes to the Greek letter  $\theta$ .

13. Construct a point  to be your independent variable. Label it  $x$ .
14. Select independent variable  $x$   and choose **Transform | Translate**.
15. Mark the translation vector by clicking in the sketch: first click parameter  $d$  and then parameter  $\theta$ . Press Translate to confirm the translation.
16. Label the new point  $T[d,\theta](x)$ .

You can read  $T_{d,\theta}(x)$  as “the translation, by distance  $d$  and angle  $\theta$ , of point  $x$ .”

17. Turn on tracing for  $x$  and  $T_{d,\theta}(x)$ . Then drag independent point  $x$ .

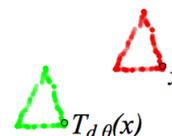
**Q5** Draw the traced shapes on your paper. How does  $T_{d,\theta}(x)$  behave?

18. Double-click one of the parameters and change its value.

19. Choose **Display | Erase Traces**, and then drag point  $x$  again.

**Q6** Draw the new traced shapes. How are they different from the ones in Q5?

- Q7** Change the parameter values so you can trace shapes arranged like the ones on the right. What parameter values did you use?



## MERGE THE POINT TO A POLYGON

Next you'll merge point  $x$  to a polygon and observe the effect on  $T_{d,\theta}(x)$ .

20. Construct an interesting polygon  with at least 5 vertices. To finish the polygon, click again on the first point.

21. To hide the vertices of the polygon, select the vertices  (but not the polygon) and choose **Display | Hide Points**.

22. Merge point  $x$  to the polygon by selecting  both the point and the polygon and choosing **Edit | Merge Point to Polygon**.

**Q8** Drag  $x$   and describe the function's domain. Where can you drag the independent variable?

A domain like this is called a *restricted domain*.

23. Select  $x$  , and choose **Edit | Action Button | Animation**. Then press OK.
24. Erase the traces and press your new *Animate Point* button.

- Q9** How does the trace of  $T_{d,\theta}(x)$  (the *range*) compare to the restricted domain? What features of the domain and range are similar, and what features are different?
- Q10** Animate  $x$  again, and describe the motion of the two points. Compare their speeds. Which point moves faster? Also compare their directions.
- Q11** Can a translation ever have any fixed points? Explain.

## A TRANSLATION CHALLENGE

25. Open **Translation Challenges.gsp** and try Challenges 1 through 7.

- Q12** Describe in step-by-step detail how you can create a new translation to exactly match any given translation. How can you find the distance, and how can you find the unknown angle? Describe any shortcuts you've invented to make it easier to find either of these features.

## Translation Challenges (short form) Name: \_\_\_\_\_

In this activity you will translate a point and compare its motion to the motion of its translated image.

### TRANSLATE A POINT

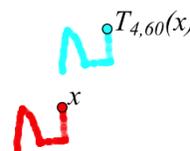
Begin by translating a point and describing how the translation behaves.

1. In a new sketch, construct an independent point  $x$  and translate it by 4 cm at  $60^\circ$ . Label the translated point  $T[4,60](x)$ , and change its color. Because point  $T_{4,60}(x)$  depends on point  $x$ , we call it the *dependent variable*.

You can read  $T_{4,60}(x)$  as “the translation of  $x$  by 4 cm at 60 degrees.”

- Q1** Drag  $x$  up. Which way does  $T_{4,60}(x)$  go? How fast does it go relative to point  $x$ ? Drag  $x$  to the left. Which way does  $T_{4,60}(x)$  go? How fast does it go?

2. Turn on tracing for points  $x$  and  $T_{4,60}(x)$ . Deselect both points and then drag independent point  $x$  to trace out an interesting shape.



- Q2** Describe the traced shapes. How are they similar, and how are they different? Consider position, size, angle, and anything else you think of. On your paper include a drawing of your two traced shapes.

- Q3** Drag again to search for fixed points. Can you find any? Explain why or why not.

- Q4** Click the **Information** tool on point  $x$ , and then on  $T_{4,60}(x)$ . How do the resulting balloons describe each point?

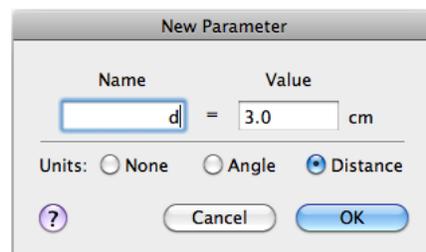
### USE AN ADJUSTABLE TRANSLATION

3. Choose **File | Document Options**. Add a new blank page, and then click **OK**.

4. Create a new distance parameter  $d = 3.0$  cm and a new angle parameter  $\{theta\} = 45^\circ$ .

5. Construct an independent point and label it  $x$ . Translate point  $x$  by the distance  $d$  and the angle  $\theta$ . Label the translated image  $T[d,\{theta\}](x)$ .

You can read  $T_{d,\theta}(x)$  as “the translation, by distance  $d$  and angle  $\theta$ , of point  $x$ .”

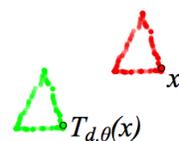


- Q5** Turn on tracing for both variables and drag independent point  $x$ . Draw the traced shapes on your paper. How does  $T_{d,\theta}(x)$  behave?

6. Change the values of the parameters, erase the traces, and then drag point  $x$  again.

- Q6** Draw the new traced shapes. How are they different from the ones in Q5?

- Q7** Change the parameter values so you can trace shapes arranged like



the ones on the right. What parameter values did you use?

## MERGE THE POINT TO A POLYGON

Next you'll merge point  $x$  to a polygon and observe the effect on  $T_{d,\theta}(x)$ .

7. Construct an interesting polygon with at least 5 vertices.
8. Hide the vertices, and merge point  $x$  to the polygon.

**Q8** Drag  $x$  and describe the function's domain. Where can you drag the independent variable?

A domain like this is called a *restricted domain*.

9. Create an animation button to animate point  $x$ .
  10. Erase the traces and press your new *Animate Point* button.
- Q9** How does the trace of  $T_{d,\theta}(x)$  (the *range*) compare to the restricted domain? What features of the domain and range are similar, and what features are different?
- Q10** Animate  $x$  again, and describe the motion of the two points. Compare their speeds. Which point moves faster? Also compare their directions.
- Q11** Can a translation ever have any fixed points? Explain.

## A TRANSLATION CHALLENGE

11. Open **Translation Challenges.gsp** and try Challenges 1 through 7.
- Q12** Describe in step-by-step detail how you can create a new translation to exactly match any given translation. How can you find the distance, and how can you find the unknown angle? Describe any shortcuts you've invented to make it easier to find either of these features.

## Translation Challenges Answers Name:

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**Q1** Drag  $x$  up. Which way does  $T_{4,60}(x)$  go? How fast does it go relative to point  $x$ ? Drag  $x$  to the left. Which way does  $T_{4,60}(x)$  go? How fast does it go?

**Q2** Describe the traced shapes. How are they similar, and how are they different? Consider position, size, angle, and anything else you think of. On your paper include a drawing of your traced shapes.

**Q3** Drag again to search for fixed points. Can you find any? Explain why or why not.

**Q4** Click the Information tool on point  $x$ , and then on  $T_{4,60}(x)$ . How do the resulting balloons describe each point?

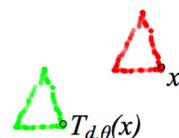
**Q5** Draw the traced shapes on your paper. How does  $T_{d,\theta}(x)$  behave?

**Q6** Draw the new traced shapes. How are they different from the ones in Q5?

## Translation Challenges Answers (continued)

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- Q7** Change the parameter values so you can trace shapes arranged like the ones on the right. What parameter values did you use?



- Q8** Drag  $x$ , and describe the function's domain. Where can you drag the independent variable?
- Q9** How does the trace of  $T_{d,\theta}(x)$  (the range) compare to the restricted domain? What features of the domain and range are similar, and what features are different?
- Q10** Animate  $x$  again, and describe the motion of the two points. Compare their speeds. Which point moves faster? Also compare their directions.
- Q11** Can a translation ever have any fixed points? Explain.
- Q12** Describe in step-by-step detail how you can create a new translation to exactly match any given translation. How can you find the distance, and how can you find the unknown angle? Describe any shortcuts you've invented to make it easier to find either of these features.

## Translation Challenges Exit Ticket Name:

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1. Describe one important thing you learned today about the translation function family.

2. Describe one thing about the translation function family that seems confusing to you.