Transform Twice (long form)   Name:

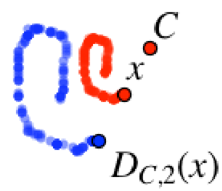
In this activity you will create a dilation function and a translation function. Then you will *compose* the functions by merging the input of the second to the output of the first.

DILATE A POINT

1. In a new sketch, construct an independent variable and label it *x*.
2. Construct a second point, label it *C*, and mark point *C* as the center for dilation.
3. Dilate point *x* by selecting it and choosing **Transform | Dilate.** The ratio is 1/2. Change it to 2/1 and click **Dilate.**
4. Use function notation to label the new dependent variable as *D[C,2](x)*.

   The label appears as $D_{C,2}(x)$. Read this as "the dilation, about *C* by a scale of 2, of variable *x*."

5. Color the new point blue, and turn on tracing for both points.



Q1 Drag independent variable *x* to make an interesting shape. On your paper draw the traces made by *x* and $D_{C,2}(x)$.

Q2 Describe the behavior of function $D_{C,2}$. What is the relative rate of change of its variables? Does it have any fixed points?

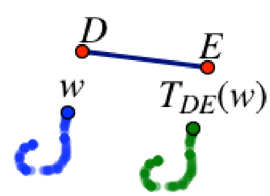
6. Erase the traces by choosing **Display | Erase Traces.**

TRANSLATE A POINT

7. Construct another independent variable point and label it *w*.
8. Construct a segment near point *w*. Label its endpoints *D* and *E*.
9. To mark the vector from *D* to *E*, select the two points in order and choose **Transform | Mark Vector.**
10. To translate point *w*, select it and choose **Transform | Translate.** The vector is from *D* to *E*. Click **Translate.**
11. Label the new dependent variable *T[DE]*(*w*).

    The label appears as $T_{DE}(w)$. Read this as "the translation, by the vector from *D* to *E*, of variable *w*."

12. Turn on tracing for *w* and $T_{DE}(w)$.



Q3 Drag independent variable *w* to make an interesting shape. On your paper draw the shapes made by *w* and $T_{DE}(w)$.

Q4 Describe the behavior of function T_{DE}. What is the relative rate of change of its variables? Does it have any fixed points?

13. Erase the traces by choosing **Display | Erase Traces.**

COMPOSE THE TRANSFORMATIONS

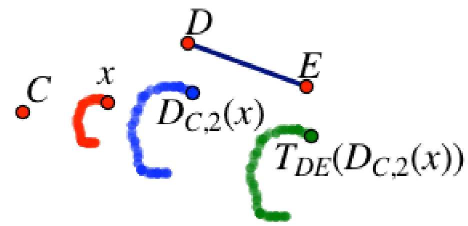
14. Select points $D_{C,2}(x)$ and *w*, and choose **Edit | Merge Points.**

Q5 Describe what happened when you chose this command.

15. The final dependent point is labeled $T_{DE}(w)$, but after merging, the point that used to be point w is now labeled $D_{C,2}(x)$. Edit the label of the final point by changing the w in the label to *D[C,2](x)*. The new label appears as $T_{DE}(D_{C,2}(x))$.

Q6 Explain why the label $T_{DE}(D_{C,2}(x))$ makes sense. Write down in words how you would read it.

16. Drag x to make an interesting shape, and observe the behavior of the two dependent points.



Q7 Draw the shapes traced by all three points.

17. Construct an interesting polygon with at least 5 vertices using the **Polygon** tool. Finish by double-clicking the last vertex.

18. Restrict independent variable x to the polygon by selecting both the point and the polygon and choosing **Edit | Merge Point to Polygon.**

Q8 Without dragging the independent variable, predict the range of the composed function. Use the **Marker** tool to draw your prediction in the sketch.

19. Erase the traces, and then animate point x by selecting it and choosing **Display | Animate Point.**

Q9 Draw the shapes made as the independent variable is animated.

Q10 Describe the traces in words. Which points are connected by $D_{C,2}$? Which points are connected by T_{DE}?

HIDE THE INTERMEDIATE VARIABLE

20. Hide point $D_{C,2}(x)$ by selecting it and choosing **Display | Hide Point.** Then erase the traces.

Q11 Animate again to observe the behavior of the composed function. Determine the relative rate of change and fixed point(s).

EXPLORE MORE

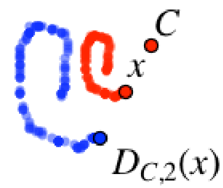
You can write the composed function using the ring symbol: $T_{DE} \circ D_{C,2}$ means “translate by vector *DE* the result of dilating about *C* by scale factor 2.”

Q12 Create a new function with the same effect as the composed function $T_{DE} \circ D_{C,2}$. From what family must it come? Determine its relative rate of change and fixed point(s), and relate them to the rate of change and fixed points of T_{DE} and $D_{C,2}$.

Transform Twice (short form)   Name:

In this activity you will create a dilation function and a translation function. Then you will *compose* the functions by merging the input of the second to the output of the first.

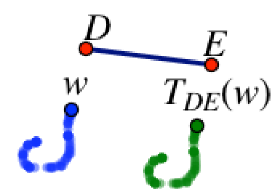
1. In a new sketch, dilate independent variable x about center C by a scale factor of 2. Use function notation to label the dependent variable, color the two variables differently, and turn on tracing for both variables.



Q1 Drag independent variable x to make an interesting shape. On your paper draw the shapes made by x and $D_{C,2}(x)$.

Q2 Describe the behavior of function $D_{C,2}$. What is the relative rate of change of its variables? Does it have any fixed points?

2. Construct segment DE and translate independent variable w by the vector from D to E. Label the dependent variable using function notation, color the variables differently, and turn on tracing.

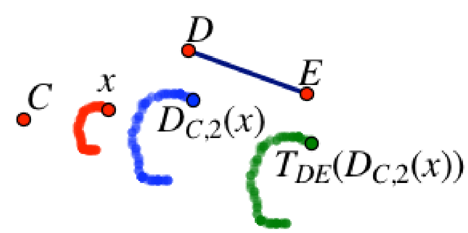


Q3 Drag independent variable w to make an interesting shape. On your paper draw the shapes made by w and $T_{DE}(w)$.

Q4 Describe the behavior of function T_{DE}. What is the relative rate of change of its variables? Does it have any fixed points?

3. Erase traces and merge the the input of T_{DE} to the output of $D_{C,2}$ by selecting them and choosing **Edit | Merge Points.**

Q5 Describe what happened when you chose this command.



4. Label the merged point $T_{DE}(D_{C,2}(x))$.

Q6 Explain why the label $T_{DE}(D_{C,2}(x))$ makes sense. Write down in words how you would read it.

5. Erase the traces and drag point x in an interesting shape.

Q7 Draw the shapes traced by all three points.

6. Construct a polygon and restrict the domain of x to the polygon.

Q8 Without dragging the independent variable, predict the range of the composed function. Use the **Marker** tool to draw your prediction in the sketch.

7. Animate point x around the domain.

You can use **Display | Animate Point,** or you can create an Animation action button.

Q9 Draw the shapes made as the independent variable is animated.

Q10 Describe the traces in words. Which points are connected by $D_{C,2}$? Which points are connected by T_{DE}?

8. Hide point $D_{C,2}(x)$ and erase the traces.

Q11 Animate again to observe the behavior of the composed function. Determine the relative rate of change and fixed point(s).

EXPLORE MORE

You can write the composed function using the ring symbol: $T_{DE} \circ D_{C,2}$ means “translate by vector *DE* the result of dilating about *C* by scale factor 2.”

Q12 Create a new function with the same effect as the composed function $T_{DE} \circ D_{C,2}$. From what family must it come? Determine its relative rate of change and fixed point(s), and relate them to the rate of change and fixed points of T_{DE} and $D_{C,2}$.

Transform Twice Answers Name:

Q1 Drag independent variable x to make an interesting shape. On your paper draw the traces made by x and $D_{C,2}(x)$.

Q2 Describe the behavior of function $D_{C,2}$. What is the relative rate of change of its variables? Does it have any fixed points?

Q3 Drag independent variable w to make an interesting shape. On your paper draw the shapes made by w and $T_{DE}(w)$.

Q4 Describe the behavior of function T_{DE}. What is the relative rate of change of its variables? Does it have any fixed points?

Q5 Describe what happened when you chose this command (**Edit | Merge Points**)?

Q6 Explain why the label $T_{DE}(D_{C,2}(x))$ makes sense. Write down in words how you would read it.

Q7 Draw the shapes traced by all three points.

Q8 Without dragging the independent variable, predict the range of the composed function. Use the **Marker** tool to draw your prediction in your sketch.

Q9 Draw the shapes made as the independent variable is animated.

Q10 Describe the traces in words. Which points are connected by $D_{C,2}$? Which points are connected by T_{DE}?

Q11 Animate again to observe the behavior of the composed function. Determine the relative rate of change and fixed point(s).

Q12 Create a new function with the same effect as the composed function $T_{DE} \circ D_{C,2}$. From what family must it come? Determine its relative rate of change and fixed point(s), and relate them to the rate of change and fixed points of T_{DE} and $D_{C,2}$.

Transform Twice Exit Ticket Name:

1. Describe in your own words what it means to compose two functions.

2. If you use the output of function f as the input to function g, and x is the input to the composed function, how would you use function notation to describe the output of the composed function? (You can write your answer using normal function notation or using the ring symbol.)