Notes for the Teacher

Students construct fractions on a number line that add up to different target sums, all less than one. Given lists of possible numerators and denominators, students must build the fractions using only those numbers. Students identify strategies for constructing the sums using the fewest possible addends.

Objectives:

- Students will use a number line model to explore adding fractions.
- Students will develop strategies for adding fractions to reach a given target sum using given numbers as choices for numerators and denominators.

Common Core Mathematical Practices: (1) Make sense of problems and persevere in solving them; (2) Reason abstractly and quantitatively; (3) Construct viable arguments and critique the reasoning of others; (5) Use appropriate tools strategically; (7) Look for and make use of structure.

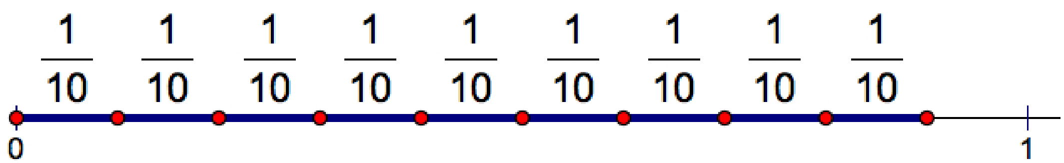
Common Core State Content Standards: 4.NF3

Grade Range: Grades 4–5

Introduce:

Use a projector to show sketch page "Game 1(A)" and distribute the worksheet. Follow worksheet step 1 to demonstrate how to make $\frac{1}{10}$. Notice that the segment of length $\frac{1}{10}$ appears after you've clicked the number 1 and clicked the number 10. You can then move the segment and click to place it so that its left endpoint coincides with 0 on the number line. (Notice that the number line will highlight in red when the segment is aligned properly.) Show students how to delete the $\frac{1}{10}$ fraction by choosing **Edit | Undo Make Fraction.**

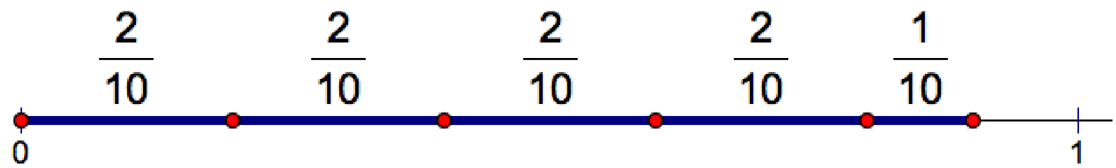
Now ask, "How can we build fractions whose sum is $\frac{9}{10}$ using the numbers 1 and 10?" Have students share their solutions. Using the given numbers, the only way is to construct $\frac{1}{10}$ nine times. Demonstrate how to build the fractions. Use the Make Fraction tool to construct segments of length $\frac{1}{10}$. Place each segment so its left endpoint coincides with the right endpoint of the previous segment. When the points highlight, click to place the segment. Be dramatic about how much time and effort it takes to build the sum of $\frac{9}{10}$ using only $\frac{1}{10}$ as a possible addend. Say, for example, "And I need to add yet another $\frac{1}{10}$, and another, and another. Wow, this is certainly taking a long time!"



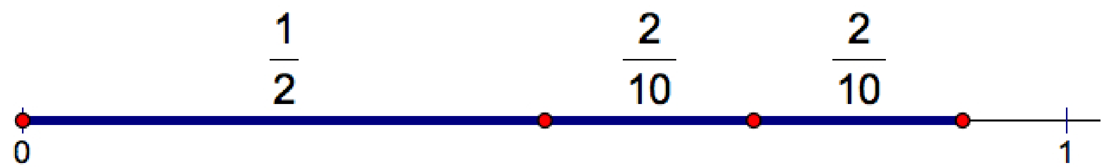
After you build the nine $\frac{1}{10}$ fractions ask, "How do you know the sum of these fractions is $\frac{9}{10}$?" Since there are nine $\frac{1}{10}$ fractions, the sum is $\frac{9}{10}$. Check your work by pressing *Show* $\frac{9}{10}$.

Ask, "What number sentence can we write to represent the fractions on the number line and their sum?" ($\frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{9}{10}$) Help students make the connection between the fraction number line model and the number sentence.

Now go to page "Game 1(B)" and ask, "How can we build fractions whose sum is $\frac{9}{10}$ using the numbers 1, 2, and 10 as numerators and denominators? Is there a quicker way than using nine $\frac{1}{10}$ fractions?" Give students some thinking time and then let volunteers share their solutions. There are several approaches they might take. Students might begin by using $\frac{1}{10}$ because that leaves a total of $\frac{8}{10}$, a fraction with an even numerator, left to build. It's easy to make fractions with even numerators: Just add together enough fractions with a numerator of 2. Thus $\frac{9}{10} = \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{2}{10} + \frac{2}{10}$. Of course, the order of the fractions ultimately does not matter. Students should realize that another valid solution is $\frac{9}{10} = \frac{2}{10} + \frac{2}{10} + \frac{2}{10} + \frac{1}{10} + \frac{2}{10}$.



It's possible, too, that some students might first make the fraction $\frac{1}{2}$, recognize that it's equal to $\frac{5}{10}$, and then build $\frac{2}{10}$ twice. Thus $\frac{9}{10} = \frac{1}{2} + \frac{2}{10} + \frac{2}{10}$.



Next go to page "Game 1(C)." This time have students find as few fractions as possible that sum to $\frac{9}{10}$ using the numbers 1, 4, and 10. Again, there are several strategies students might take, and they're all worth exploring as a class. For example, students might begin by subtracting $\frac{1}{10}$ from $\frac{9}{10}$ and realize that the remainder, $\frac{8}{10}$, can be split into two equal fractions that are equivalent to $\frac{4}{10}$. Thus $\frac{9}{10} = \frac{1}{10} + \frac{4}{10} + \frac{4}{10}$. Alternatively, students might begin by forming $\frac{4}{10}$, leaving just $\frac{5}{10}$, or $\frac{1}{2}$ remaining. One half can be split into two fractions, each with value $\frac{1}{4}$. Thus $\frac{9}{10} = \frac{4}{10} + \frac{1}{4} + \frac{1}{4}$. Both of these methods share the same basic strategy: Subtract enough from 9/10 to leave a fraction that can be spit evenly in half. These kinds of observations—noting the similarities and differences in students' strategies for reaching the target fraction—are worth discussing when students come back together as a whole class.

Explore:

Assign students to partners and send them in pairs to the computers.

Tell students that they will play two similar games, finding how to use as few fractions as possible to reach a target sum. Ask students to start on page "Game 2(A)" and then continue through page "Game 3(C)."

As you circulate, observe students as they work. Make sure students record the number sentences and their explanations on the worksheet.

Discuss:

Call students together to discuss and summarize what they learned. Ask them to share the strategies they used to find the most efficient way to reach the target sums. Here are some sample replies:

- *We saw that a numerator of 1 would require using the most fractions, so we steered clear of using 1 as a numerator when possible.*
- *We built fractions with as large a denominator as possible so that we could reach the sum as quickly as possible. Sometimes we had to delete a fraction with a large numerator because it went over the sum. Then we had to use a smaller numerator until we reached the sum.*
- *We thought about what numbers we could add to reach the numerator in the sum. We repeatedly added the largest number without going over the numerator in the sum. Then we added smaller numbers. For example, for* $\frac{30}{37}$, *we added 13 + 13 to get 26 and then added 4 to get 30. So, our number sentence was* $\frac{13}{37} + \frac{13}{37} + \frac{4}{37} = \frac{30}{37}$.

Sketch pages "Make Your Own (A)," "Make Your Own (B)," and "Make Your Own (C)" allow students to create their own games, similar to the ones they just played. First, students should pick a fraction as their target sum. They can enter this target into the sketch by replacing the question marks on the Make Your Own pages with the values of the numerator and denominator for their target fraction.

Then, students should decide which numbers will be available to use as numerators and denominators. (As a default, the sketch pages list the numbers 1, 1, 1, and 1, but students can use any numbers they like. They can also choose to delete one or more of the 1s if they decide to offer fewer choices.) To change a 1 to a different value, students should double-click the number with the **Arrow** tool, enter a new value, and click **OK.**

Have students share their games with the entire class to solve.

Related Activities:

- *Time-Saver Games, Part Two—Adding Fractions*
- *Fractions on a Number Line—Sums of One*

- *Fractions on a Number Line—Addition and Subtraction Games*
- *Fractions on a Number Line—Adding with Unlike Denominators*
- *Measuring with Fractions— Fractions on a Number Line*

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Portions of this material are based upon work supported by the National Science Foundation under award number DRL-0918733. Any opinions, findings, and conclusions or recommendations expressed in this work are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.