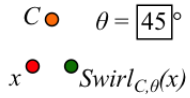
Special Effects (long form) Name:

In this activity you will create a geometric function that produces a swirling effect. You'll define the function using point variables and then apply it to other geometric objects.

WARM-UP: A ROTATION FUNCTION

1. In a new sketch, construct a center point *C* and an angle parameter $\theta = 45°$. Type {theta} for the name of the parameter, and Sketchpad will change the name to θ.
2. Construct independent variable *x* and choose **Transform | Rotate.** With the Rotate dialog box open, click *C* in the sketch to mark it as the center, and click θ to mark it as the angle of rotation. Then click Rotate.
3. Label the rotated point *Swirl[C,θ](x)*.
4. Turn on tracing for the independent and dependent variables.



Q1 Drag the independent variable and describe the behavior of the function.

Q2 Change the parameter's value to 15° and drag *x* again. How does the new angle change the behavior of the function?

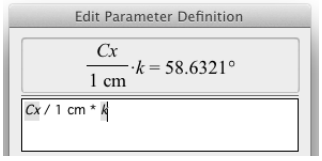
A MORE INTERESTING FUNCTION

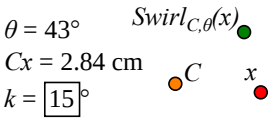
Now make the rotation depend on the distance from the center.

5. Measure the distance *Cx* from the center to the independent variable.

   Select the two points and choose **Measure | Distance.**

6. Select parameter θ and choose **Edit | Edit Parameter.** Use the Calculator to calculate an angle that depends on the distance measurement. To start, delete the current value (15°), click the distance measurement *Cx*, and divide the distance measurement by 1 cm. Then use the Calculator's Value menu to choose **New Parameter,** naming it *k* and giving it a value of 15°. Finally click OK.





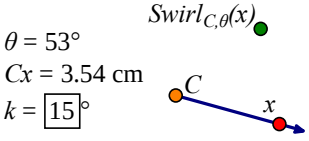
Q3 Using this calculation as your angle of rotation, what do you predict the angle of rotation will be when *x* is 3 cm from *C*? What will it be when *x* is 6 cm from *C*?

Q4 Drag *x* toward and away from the center to check your predictions. What do you notice about the motion of $Swirl_{C,\theta}(x)$?

7. Save your sketch to use in the next activity "Swirl a Picture."

A RESTRICTED DOMAIN

8. Use the **Ray** tool to construct a ray starting from point *C,* and merge the independent variable to the ray.



Q5 Drag point x along the domain. Describe the range that corresponds to this restricted domain.

9. Make the range a permanent object by constructing the locus of $Swirl_{C,\theta}(x)$ as x moves along the ray.

   To construct the locus, select x and $Swirl_{C,\theta}(x)$, and choose **Construct | Locus.**

10. Drag the independent variable along the domain to verify that the locus really is the range of the function.

Q6 Draw the range on your paper, and explain why it has the shape that it does.

A FAMILY OF SWIRLING FUNCTIONS

Q7 What do you think would happen if you change the value of k? Write down what value of k you will try first, and write down your prediction.

Q8 Test your prediction by double-clicking k and changing its value. Draw and describe your results. Was the effect what you predicted?

Q9 Predict what will happen if you make the value of k negative. Then test your prediction, and draw and describe the result.

Q10 What can you say about the relative rate of change of x and $Swirl_{C,\theta}(x)$? Identify any fixed points that you can locate.

Q11 Create an animation action button to animate k bidirectionally between $-90°$ and $90°$. Press the button, describe the result, and explain how this special effect relates to the function you created in step 6.

To create the button, select k and choose **Edit | Action Buttons | Animation.**

EXPLORE MORE

Q12 Try setting $k = 120°$. Can you find fixed points, and if so, where are they?

Q13 Leave $k = 120°$, and move x to a distance of 6 cm from point C. What are the relative speeds and directions of the variables for this location of x? Be as precise as you can, and explain your reasoning.

Q14 Split the independent variable from the ray, and restrict it to a polygon. Construct the range and compare it to the domain. Are they similar and/or congruent? Draw them, and explain the shape of the range.

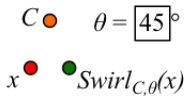
Q15 Create a new function that depends on some other characteristic of the independent variable. What if the scale factor of a dilation depends on the distance from the center point, or on the angle the independent variable makes with the center point? What if a translation or reflection depends on the distance of the independent variable from a particular line? Use your imagination!

Special Effects (short form) Name:

In this activity you will create a geometric function that produces a swirling effect. You'll define the function using point variables and then apply it to other geometric objects.

EXPLORE A FUNCTION

1. In a new sketch, construct the rotation of independent variable *x* around center point *C* by an angle parameter $\theta = 45°$. Label the dependent variable $Swirl_{C,\theta}(x)$ and turn on tracing for both variables.

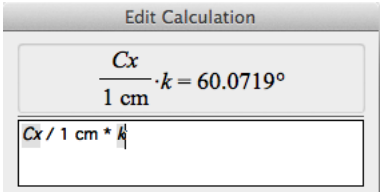


Q1 Drag the independent variable and describe the behavior of the function.

Q2 Change θ to 15°. How does the new angle change the function's behavior?

Now make the rotation depend on the distance from the center.

2. Measure the distance *Cx* from the center to the independent variable.
3. Select parameter θ and edit the parameter, using the Calculator to perform the computation shown in the picture. To create the value *k,* use the Calculator's Value menu to choose **New Parameter,** name it *k* and give it a value of 15°.



Q3 Using this calculation as your angle of rotation, what do you predict the angle of rotation will be when *x* is 3 cm from *C*? What will it be when *x* is 6 cm from *C*?

Q4 Drag *x* toward and away from the center to check your predictions. What do you notice about the motion of $Swirl_{C,\theta}(x)$?

4. Save your sketch to use in the next activity "Swirl a Picture."
5. Restrict the domain to a ray that starts at point *C*.

Q5 Drag point *x* along the domain. Describe the corresponding range.

6. Make the range a permanent object by constructing the locus of $Swirl_{C,\theta}(x)$ as *x* moves along the ray. Drag the independent variable along the domain to verify that the locus is the range of the function.

Q6 Draw the range on your paper, and explain why it has the shape that it does.

Q7 What do you think would happen if you change the value of *k*? Write down what value of *k* you will try first, and write down your prediction.

Q8 Test your prediction by changing the value of *k*. Draw and describe your results. Was the effect what you predicted?

Q9 Predict what will happen if you make the value of k negative. Then test your prediction, and draw and describe the result.

Q10 What can you say about the relative rate of change of x and $Swirl_{C,\theta}(x)$? Identify any fixed points that you can locate.

Q11 Create an animation action button to animate k bidirectionally between $-90°$ and $90°$. Press the button, describe the result, and explain how this special effect relates to the function you created in step 3.

EXPLORE MORE

Q12 Try setting $k = 120°$. Can you find fixed points, and if so, where are they?

Q13 Leave $k = 120°$, and move x to a distance of 6 cm from point C. What are the relative speeds and directions of the variables for this location of x? Be as precise as you can, and explain your reasoning.

Q14 Split the independent variable from the ray, and restrict it to a polygon. Construct the range and compare it to the domain. Are they similar and/or congruent? Draw them, and explain the shape of the range.

Q15 Create a new function that depends on some additional characteristic of the independent variable. What if the scale factor of a dilation depends on the distance from the center point, or on the angle the independent variable makes with the center point? What if a translation or reflection depends on the distance of the independent variable from particular line? Use your imagination!

Special Effects Answers Name:

Q1 Drag the independent variable and describe the behavior of the function.

Q2 Change the parameter's value to 15° and drag x again. How does the new angle change the behavior of the function?

Q3 Using this calculation as your angle of rotation, what do you predict the angle of rotation will be when x is 3 cm from C? What will it be when x is 6 cm from C?

Q4 Drag x toward and away from the center to check your predictions. What do you notice about the motion of $Swirl_{C,\theta}(x)$?

Q5 Drag point x along the domain. Describe the range that corresponds to this restricted domain..

Q6 Draw the range on your paper, and explain why it has the shape that it does.

Q7 What do you think would happen if you change the value of k? Write down what value of k you will try first, and write down your prediction.

Q8 Test your prediction by changing the value of k. Draw and describe your results. Was the effect what you predicted?

Q9 Predict what will happen if you make the value of k negative. Then test your prediction, and draw and describe the result.

Q10 What can you say about the relative rate of change of x and $Swirl_{C,\theta}(x)$? Identify any fixed points that you can locate.

Q11 Create an animation action button to animate k between −90° and 90°. Press the button, describe the result, and explain how this special effect relates to the function you created in step 6 (long form) or step 3 (short form).

EXPLORE MORE

Q12 Try setting $k = 120°$. Can you find fixed points, and if so, where are they?

Q13 Leave $k = 120°$, and move x to a distance of 6 cm from point C. What are the relative speeds and directions of the variables for this location of x? Be as precise as you can, and explain your reasoning..

Q14 Split the independent variable from the ray, and restrict it to a polygon. Construct the range and compare it to the domain. Are they similar and/or congruent? Draw them, and explain the shape of the range.

Q15 Create a new function that depends on some additional characteristic of the independent variable. What if the scale factor of a dilation depends on the distance from the center point, or on the angle the independent variable makes with the center point? What if a translation or reflection depends on the distance of the independent variable from particular line? Use your imagination! Submit your finished sketch.

Special Effects Exit Ticket Name:

1. Describe something that surprised or puzzled you as you did this activity. How is this family of swirling functions similar to other function families, and how is it different?

2. How is this family of swirling functions similar to other function families, and how is it different?

