Rotation Challenges (long form)　Name:

In this activity you will rotate a point and compare its motion to the motion of its rotated image.

## ROTATE A POINT

Begin by rotating a point and describing how the rotation behaves.

1. In a new sketch, construct an independent point $\boxed{\cdot}$ and label it x $\boxed{\mathbf{A}}$.

2. Drag the point $\boxed{\text{↖}}$ around in your sketch.
   Because you can drag this point anywhere in your sketch, and it doesn't depend on other objects, we say that point x is the *independent variable.*

3. Construct another point $\boxed{\cdot}$ in the center of the sketch. Label it C $\boxed{\mathbf{A}}$.

4. With point C selected, choose **Transform | Mark Center.**

5. Select point x $\boxed{\text{↖}}$ and choose **Transform | Rotate.** Click the Rotate button to create the rotated image.

6. Label the rotated point $\boxed{\mathbf{A}}$ *R[C,90](x)*. Because point $R_{C,90}(x)$ depends on point $x,$ we call it the *dependent variable.*

   You can read $R_{C,90}(x)$ as "the rotation of x about C by 90°."

7. Change the color of the dependent point $R_{C,90}(x)$ by selecting it $\boxed{\text{↖}}$ and choosing **Display | Color.**

Q1 Drag x up. Which way does $R_{C,90}(x)$ go? Drag x left. Which way does $R_{C,90}(x)$ go?

8. Turn on tracing for points x and $R_{C,90}(x)$, by selecting them both $\boxed{\text{↖}}$ and choosing **Display | Trace Points.**

9. Click in empty space $\boxed{\text{↖}}$ to deselect both points. Then drag independent point x to trace out an interesting shape.
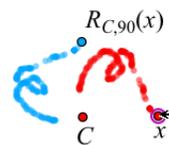
Q2 Describe the traced shapes. How are they similar, and how are they different? Consider position, size, angle, and anything else you think of. On your paper include a drawing of your two traced shapes.

10. Erase the traces by choosing **Display | Erase Traces,** and then trace a new shape. Make this shape go through the center point.

Q3 What happened when you went through the center of rotation? Describe these traced shapes, and include a drawing on your paper. Were there any fixed points? If so, where?

   A location where x and $R_{C,90}(x)$ come together is called a *fixed point* of the function.

Q4 Click the **Information** tool $\boxed{\text{ⓘ}}$ on point $x,$ and then on $R_{C,90}(x)$. How do the resulting balloons describe each point?

USE A DIFFERENT ANGLE OF ROTATION

11. Choose **File | Document Options** and add a new page to your document.

    In the dialog box, press the Add Page button. Choose **Blank Page** and then press OK.

12. On the new page, choose **Number | New Parameter.** Change the label to {theta}, set the value to 45, and set the Units radio button to Angle.

    When you type {theta}, Sketchpad changes the label to θ.

13. As you did before, construct ⚫ independent variable x and point C, and mark point C as the center for rotation.

14. Select point x 🖱 and choose **Transform | Rotate.** With the Rotate dialog box open, click angle parameter θ. Then click Rotate to finish the rotation.

15. Label the rotated point Ⓐ $R[C,\{theta\}](x)$.

Q5 Drag x up. Which way does $R_{C,\theta}(x)$ go? Drag x left. Which way does $R_{C,\theta}(x)$ go?

Q6 Can you drag x on top of $R_{C,\theta}(x)$ to find a *fixed point*? If so, where is it?

16. Turn on tracing 🖱 for both points, and drag x to make a shape.

Q7 Describe and draw the new traced shapes. How are these shapes different from the shapes you made when you rotated by 90°? Do your shapes have any fixed points?

17. Double-click θ 🖱, change its value to 180°, and erase the traces.

Q8 Drag x 🖱 in different directions. Which way does $R_{C,\theta}(x)$ go when you drag x up? Which way does $R_{C,\theta}(x)$ go when you drag x right? Can you find any fixed points?

RESTRICT THE DOMAIN

Now you'll merge point x to a polygon and observe the effect on $R_{C,\theta}(x)$.

18. Construct an interesting polygon 🔷 with at least 5 vertices. To finish the polygon, click again on the first point.

19. To hide the vertices of the polygon, select the vertices 🖱 (but not the polygon) and choose **Display | Hide Points.**

20. Merge point x to the polygon by selecting both the point and the polygon and choosing **Edit | Merge Point to Polygon.**

Q9 Drag x, and describe the function's domain. In other words, where can you drag x?

    A domain like this is called a *restricted domain*.

21. Select x, and choose **Edit | Action Button | Animation.** Then press OK.

22. Erase the traces and press your new *Animate Point* button.
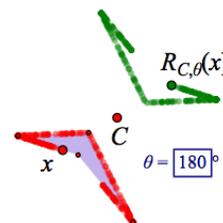
Q10 How does the trace of $R_{C,\theta}(x)$ (the *range*) compare to the restricted domain? What features of the domain and range are similar, and what features are different?

A ROTATION CHALLENGE

Now you'll look at some rotations to locate the invisible center of rotation and find the unknown angle of rotation.

23. Open **Rotation Challenges.gsp** and try the Challenges.

Q11 Describe in step-by-step detail how you can create a new rotation to exactly match any given rotation. How can you locate the invisible center of rotation, and how can you find the unknown angle? Describe any shortcuts you've invented to make it easier to find either of these features.

Rotation Challenges (short form)   Name:

In this activity you will rotate a point and compare its motion to the motion of its rotated image.

ROTATE A POINT

Begin by rotating a point and describing how the rotation behaves.

1. In a new sketch, construct an independent point and label it x.

2. Construct another point in the center of the sketch. Label it C, and mark it as the center.

3. Rotate x about center point C. Label the rotated point $R[C,90](x)$, and change its color. Because point $R_{C,90}(x)$ depends on point x, we call it the *dependent variable*.

   You can read $R_{C,90}(x)$ as "the rotation of x about C by 90°."

Q1 Drag x up. Which way does $R_{C,90}(x)$ go? Drag x left. Which way does $R_{C,90}(x)$ go?

4. Turn on tracing for points x and $R_{C,90}(x)$. Deselect both points and then drag independent point x to trace out an interesting shape.
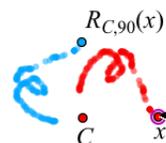
Q2 Describe the traced shapes. How are they similar, and how are they different? Consider position, size, angle, and anything else you think of. On your paper include a drawing of your two traced shapes.

5. Erase the traces and trace a new shape that goes through the center point.

Q3 What happened when you went through the center of rotation? Describe these traced shapes, and include a drawing on your paper.

   A location where x and $R_{C,90}(x)$ come together is called a *fixed point* of the function.

   Were there any fixed points? If so, where were they?

Q4 Click the **Information** tool on point x, and then on $R_{C,90}(x)$. How do the resulting balloons describe each point?

USE A DIFFERENT ANGLE OF ROTATION

6. In a new sketch, create a new parameter. Label it {theta}, set its value to 45, and set the Units radio button to Angle.

   When you type {theta}, Sketchpad changes the label to θ.

7. Construct independent variable x and point C.

8. Rotate x about center point C. Label the rotated point $R[C,\{theta\}](x)$, and change its color. Point $R_{C,\theta}(x)$ is the *dependent variable*.

   If Sketchpad doesn't automatically mark C as the center, click C while the Rotate dialog box is open.

Q5 Drag x up. Which way does $R_{C,\theta}(x)$ go? Drag x left. Which way does $R_{C,\theta}(x)$ go?

**Q6** Can you drag x on top of $R_{C,\theta}(x)$ to find a *fixed point*? If so, where is it?

9. Turn on tracing for both points, and drag *x* to make a shape.

Q7 Describe and draw the new traced shapes. How are these shapes different from the shapes you made when you rotated by 90°? Do your shapes have any fixed points?

10. Double-click *θ*, change its value to 180°, and erase the traces.

Q8 Drag *x* in different directions. Which way does $R_{C,\theta}(x)$ go when you drag *x* up? Which way does $R_{C,\theta}(x)$ go when you drag *x* right? Can you find any fixed points?

RESTRICT THE DOMAIN

Now you'll merge point *x* to a polygon and observe the effect on $R_{C,\theta}(x)$.

11. Construct an interesting polygon with at least 5 vertices, hide the vertices, and merge point *x* to the polygon.

Q9 Drag *x,* and describe the function's domain. In other words, where can you drag the independent variable?

A domain like this is called a *restricted domain*.

12. Erase the traces, and create an Animation button to animate independent variable *x* around the polygon. Press your new *Animate Point* button.

Q10 How does the trace of $R_{C,\theta}(x)$ (the *range*) compare to the restricted domain? What features of the domain and range are similar, and what features are different?
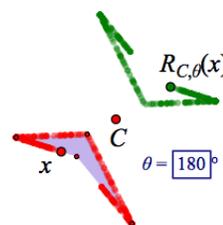
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Rotation Challenges Answers   Name:

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Q3 What happened when you went through the center of rotation? Describe these traced shapes, and include a drawing on your paper.

Q4 Click the **Information** tool on point x, and then on $R_{C,90}(x)$. How do the resulting balloons describe each point?

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Rotation Challenges Exit Ticket   Name:

1. Describe one important thing you learned today about the rotation function family.

2. Describe one thing about the rotation function family that seems confusing to you.