Notes for the Teacher

Students develop their estimation skills to identify "mystery" fractions on a 0–1 number line. Each mystery fraction is an unlabeled point on a number line whose location can be described as fraction. The goal is to name the exact location in as few tries as possible. Students estimate the location of the point, vew their estimate on the number line, and then use their estimate to make a new estimate. Through repeated estimates, students are able to pinpoint the mystery fraction's location.

Objectives:

- Students will use a number line model to explore fractions.
- Given an unlabeled tick mark on a number line, students use visual estimation skills as well as the location of previous guesses to name the exact location of the tick mark as a fraction.

Common Core Mathematical Practices: (1) Make sense of problems and persevere in solving them; (2) Reason abstractly and quantitatively; (3) Construct viable arguments and critique the reasoning of others; (5) Use appropriate tools strategically; (7) Look for and make use of structure.

Common Core State Content Standards: 3.NF1, 2; 4.NF3

Grade Range: Grades 3–5

Introduce:

Open **Mystery Fractions Part Two--Estimating and Refining.gsp** and distribute the worksheet. Use a projector to show sketch page "Game A." Explain that the goal is to identify the location of the point on the green tick mark on the number line in as few guesses as possible. Tell students that the location of the point can be expressed as a fraction whose numerator and denominator are both between 1 and 12.

The description that follows applies to the initial fraction displayed on the number line. To get started, demonstrate how to mark the location of $\frac{1}{2}$ on the number line by following worksheet step 1. Choose the **Make Fraction** tool from the **Custom Tools** menu. Click the numerator, 1, and then the denominator, 2. The labeled fraction for $\frac{1}{2}$ will appear on the number line.

Ask, "Is the mystery fraction greater than or less than $\frac{1}{2}$?" (Yes) Ask, "What are some reasonable predictions?" As a class, make a list of possible fractions on the board using the available values for numerator and denominator. Students should use their estimation skills to make predictions. They may say that fractions like $\frac{3}{4}$ and $\frac{4}{5}$ seem too big because they are close to 1. Fractions like $\frac{2}{3}$, $\frac{3}{5}$, $\frac{4}{7}$, $\frac{5}{8}$, $\frac{5}{9}$, and $\frac{6}{11}$ are more reasonable choices. Have students justify why a particular fraction is a reasonable estimate. For example, a student may suggest $\frac{4}{7}$ because it is a little more than $\frac{4}{8}$, or $\frac{1}{2}$. Another student may propose 3/5 because it is equivalent to 0.6, which is just 0.1 more than 0.5.

Have a volunteer make a fraction from the list and compare it to the mystery fraction. If the selection is not the mystery fraction, ask "Is the fraction too large or too small? What should we try next?" Continue to let volunteers try fractions, using the guesses to narrow the choices. Have students explain their reasoning when selecting a subsequent fraction to try. After the mystery fraction is found ($\frac{3}{5}$), press the *Show Answer* button with the **Arrow** tool to check.

Demonstrate how to move on to the next challenge. Go to page "Game B." Press *New Problem* to generate a new random Mystery Fraction challenge to solve.

Explore:

Assign students to partners, and send them in pairs to the computers. Have students open **Mystery Fractions Part Two--Estimating and Refining.gsp**.

Tell students that they will use the fraction tool to find the location of each mystery fraction in as few guesses as possible. As you circulate, notice the strategies students use to identify the mystery fractions. Do students make a list of reasonable fractions to try? Do they narrow their choice of fractions in the list by using their previous guesses? Make sure students record all guesses, the mystery fractions, and their explanations on the worksheet.

Discuss:

Call students together to discuss and summarize what they've learned. Review the solutions. Have students share strategies for finding the mystery fractions. The next page gives some possible student strategies.

- (The value of the mystery fraction is $\frac{1}{5}$.) *We used halves, quarters, and eighths as benchmarks. We saw that the mystery fraction was clearly less than* $\frac{1}{2}$. *It looked like it was less than* $\frac{1}{4}$ *but more than* $\frac{1}{8}$. *We didn't construct either* $\frac{1}{4}$ *or* $\frac{1}{8}$ *on the number line, but we could have to be sure our visual estimates were correct. Since we estimated that the mystery fraction was larger than* $\frac{1}{8}$, *we guessed* $\frac{1}{6}$. *It was too small, so then we guessed* $\frac{1}{5}$. *We were right!*

- (The value of the mystery fraction is $\frac{11}{12}$.) *For some problems, we used the distance between the mystery fraction and the closest endpoint as a 'unit.' Then we estimated how many of those units fit between 0 and 1. That gave us our denominator. Then we estimated how many units there were from 0 to the mystery fraction. Here, for example, the distance from the mystery fraction to 1 became our unit. It looked like 10 of those units fit between 0 and 1, and it looked like the mystery was at* $\frac{9}{10}$. *So, we guessed* $\frac{9}{10}$. *It was too small. Then we guessed* $\frac{10}{11}$. *Still too small. Finally, we guessed* $\frac{11}{12}$ *and got it right.*

- (The value of the mystery fraction is $\frac{4}{7}$.) *We immediately estimated if the mystery fraction was less than or greater than* $\frac{1}{2}$. *That made it easier for us to divide just half the number line into equal parts. Here, for example, we could see that the mystery fraction was greater than* $\frac{1}{2}$. *It looked like the distance between* $\frac{1}{2}$ *and the mystery fraction was one-sixth the distance from* $\frac{1}{2}$ *to 1, or equivalently one-twelfth the distance from 0 to 1. So we guessed that the mystery fraction was* $\frac{1}{2} + \frac{1}{12}$, *or* $\frac{7}{12}$. *That was too large, but it was very close.*

- *If the mystery fraction was fairly close to 0, we reasoned that the numerator had to be 1. Then we imagined dividing the number line into equal parts, using*

the distance from 0 to the mystery fraction as a 'unit.' That gave us an estimate of the denominator. Similarly, if the mystery fraction was close to 1, we figured the numerator was probably one less than the denominator.

Explore More:

Students will enjoy making their Mystery Fraction challenges for each other to solve on page "Make Your Own." While students work in pairs, one should looks away while the other student enters values for the numerator and denominator of the mystery fraction. After this student has entered the fraction, she should press *Hide Fraction* and allow her partner to solve the challenge. The students continue by switching roles and playing again.

Note that the well of numbers in the top-right corner of the sketch starts, by default, with the values 1 through 12, but these can be changed by double-clicking any number and entering a new value. Doing so allows students (or the teacher) to create more varied challenges.

Related Activities:

- *Mystery Fractions, Part One—Estimating and Refining*

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