Notes for the Teacher

In this activity, students use a rectangular array model to develop strategies for decomposing multiplication problems into simpler problems with numbers whose products they already know. By practicing with the array models, students increase their fluency with multiplication facts for 6's, 7's, 8's, 9's, 11's, and 12's.

Objectives:

- Students will use a rectangular array model to explore multiplication facts for 6's, 7's, 8's, 9's, 11's, and 12's.
- Students will see the relationship between a multiplication fact represented in numerical form and the same fact represented with an array.
- Students will develop and use strategies such as the distributive property for computing multiplication facts.

Common Core Mathematical Practices: (1) Make sense of problems and persevere in solving them; (2) Reason abstractly and quantitatively; (3) Construct viable arguments and critique the reasoning of others; (5) Use appropriate tools strategically; (7) Look for and make use of structure.

Common Core State Content Standards: 3.OA3, 5, 7, 9

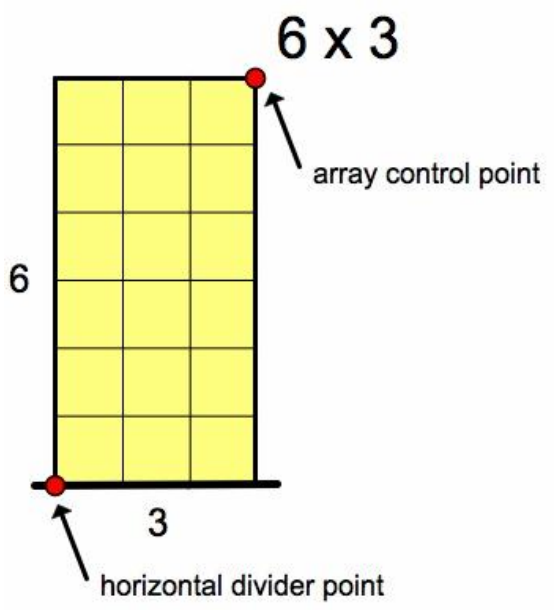
Grade Range: Grades 3–4

Introduce:

Open **Multiplication Strategies--Multipliers 6 Through 12.gsp,** and distribute the worksheet. Use a projector to show page "Array", "Circles", or "Circles 2." All three models represent multiplication using an array model, but both Circles pages represents multiplication with an array of discrete objects (Page "Circles" shows an array of green circles; Page "Circles 2" shows an array of orange and brown circles.) Younger students may find the circles models easier to understand.

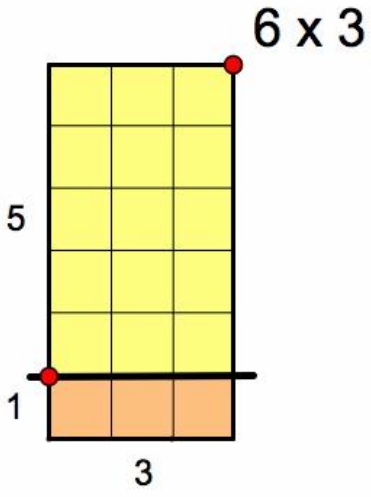
Sixes

Use the **Arrow** tool to select the red point on the horizontal divider and drag it all the way down.



Define *array* if students have not encountered this term before. Explain that an array is an arrangement of objects in rows and columns that makes the objects easier to count. Tell students that this is an array of squares that form a rectangle. Ask, "How many rows are there in the array? How many columns?" (6 rows and 3 columns) Explain that arrays can be used to show multiplication facts. Ask, "How does this array show 6 × 3?" Students should reply that there are 6 identical rows with 3 squares in each row or 3 equal columns with 6 squares in each column. (Note: For this activity, the first number in each multiplication statement is the number of rows and the second number is the number of columns. Establish this convention with students when discussing the arrays so everyone understands what is being described.)

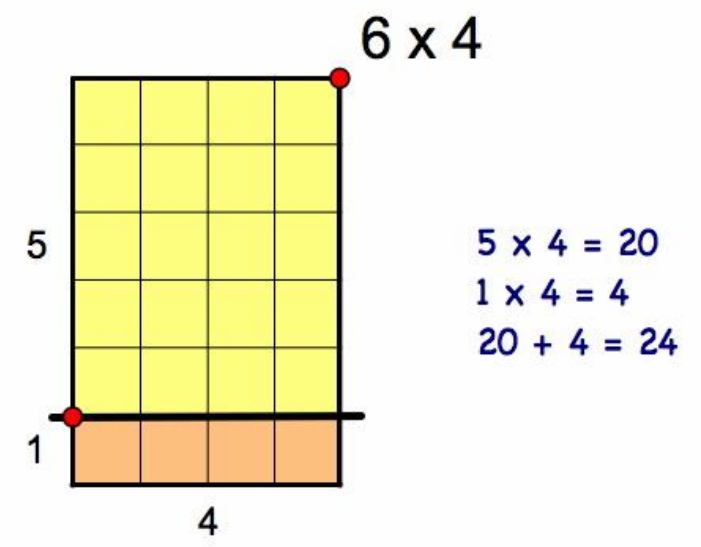
Use the **Arrow** tool to drag the vertical divider up one row so that the array is divided as shown below.



Explain that the divider now splits the 6 × 3 array into two smaller arrays. Ask, "What multiplication fact is shown by the yellow array?" (5 × 3) Write 5 × 3 on the board. Ask, "What multiplication fact is shown by the orange array?" (1 × 3) Write 1 × 3 on the board.

Ask, "What is 5 × 3?" (15) "What is 1 × 3?" (3) The 5's facts and 1's facts are easy for students to remember, so they should know these facts already. "What is 15 + 3?" (18) "So, what is 6 × 3?" (18)

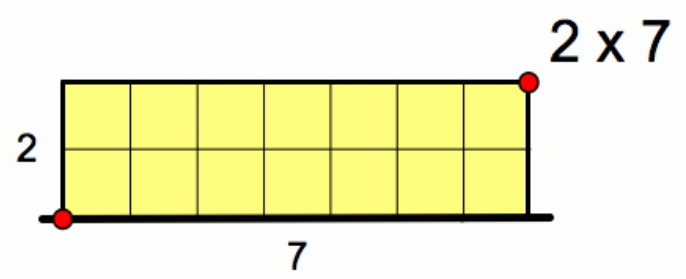
Now form a new multiplication problem by dragging the array control point up to show 6 × 4. Ask, "What multiplication fact are we showing now?" (6 x 4). "This array is split into two smaller arrays. What multiplication fact is shown by the yellow array? The orange array?" (5 × 4 and 1 × 4) Have students find 6 × 4 by computing the number of squares in the yellow and orange arrays and adding the two numbers together.



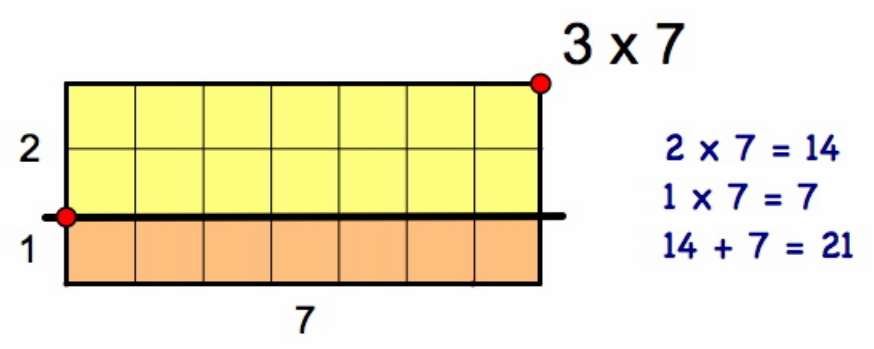
Do several more problems with the 6's facts (e.g., 6 × 6, 6 × 7, 6 × 8, 6 × 12). Then ask, "Why does dividing a 6's fact into a 5's fact and a1's fact help you remember the 6's fact?" Students may reply, "Multiplying by 5 and by 1 is easy. I can do those facts in my head and then add them together to answer the times 6 fact."

Sevens

Use the **Arrow** tool to drag the vertical divider and the grid control point to show 2 × 7. Ask, "What is 2 × 7?" (14) Students should have easily mastered the 2's facts already.



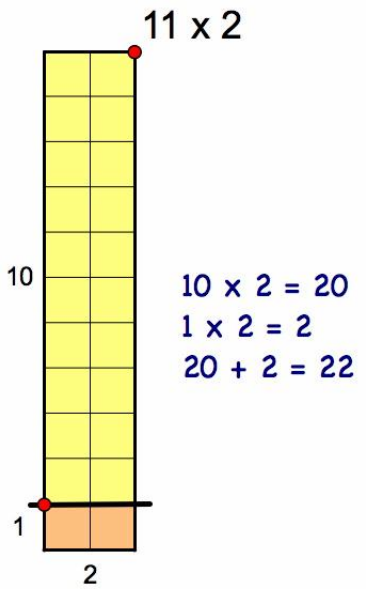
Now show 3 × 7 divided into two smaller arrays: 2 × 7 and 1 × 7. Ask, "How can you use what you know about 2 x 7 to find 3 x 7?" Students may reply, "To find 3 × 7, we start by thinking about 2 × 7. We already know that 2 x 7 is 14. So all we need to do is add on another 7. It is 14 + 7, or 21." Explain that this is known as an "adding on" strategy.



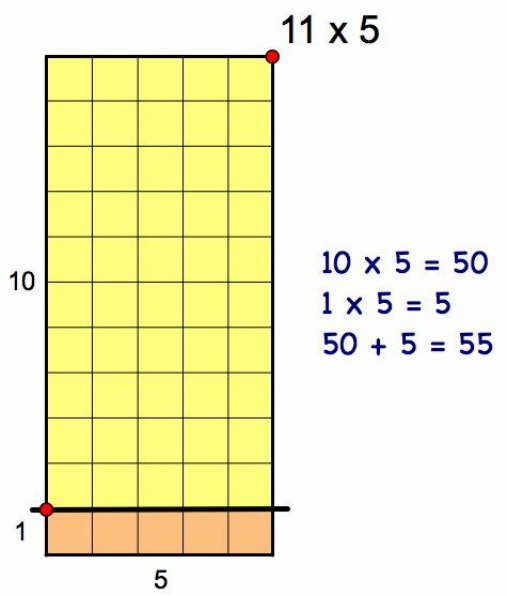
Continue in this manner, showing 4 × 7 composed of 3 × 7 (which students now know) plus another 7. Work through the remaining 7's facts using the "adding on" strategy.

Elevens and Twelves

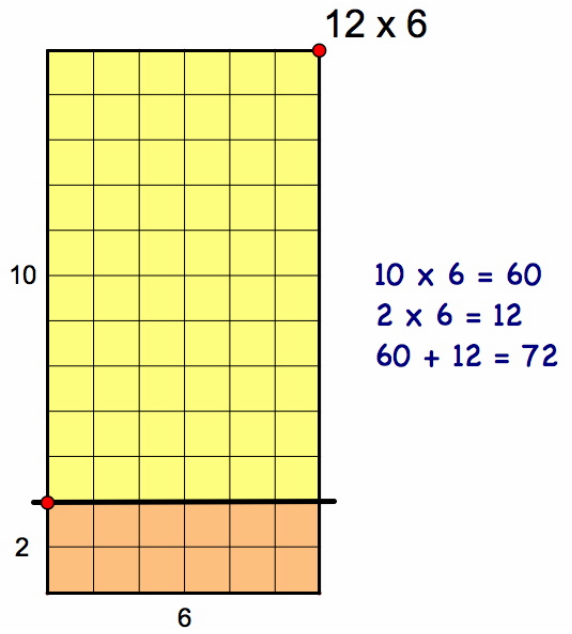
Display 11 × 2 as two smaller arrays: 10 × 2 and 1 × 2. Ask, "How does breaking an 11's fact into a 10's fact and a 1's fact make remembering the 11's facts easier?" A student may reply, "I can do 10's and 1's facts in my head already. I just need to add them to find the 11's fact." Write the equations on the board so students see the connection to the model.



Have students find the answers to the other 11's facts, such as 11 × 5, or 11 ×x 11 in a similar manner.



Then use the 10's and 2's facts to compute the 12's facts with students. For example, 12 × 6 can be divided into two smaller arrays: 10 × 6 and 2 × 6.



Explore:

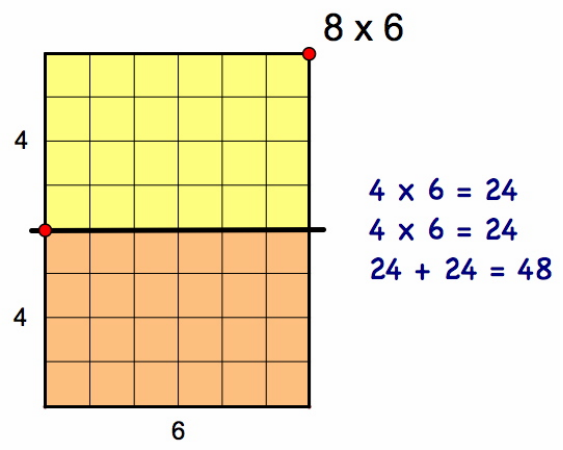
Assign students to partners and send them in pairs to the computers. Have students open **Multiplication Strategies--Multipliers 6 Through 12.gsp** and go to page "Array" or "Circles" depending on which array representation you wish to use. Ask students to develop strategies for learning the 8's and 9's facts. Make sure students understand how to record the results on the worksheet.

As you circulate, observe students as they work. What strategies do students use to find the 8's and 9's facts? Are students splitting the array for 8's facts in half and solving two 4's facts? Are they using 5's facts as one of the smaller arrays to solve 8's and 9's facts? Listening to students as they create strategies will give you insight into their understanding of numbers and operations.

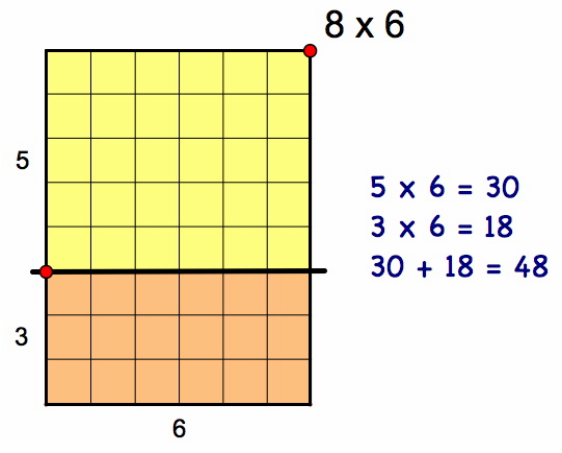
Discuss:

Call students together to discuss the different strategies they developed for finding the 8's and 9's facts. Have students demonstrate their strategies on the computer and write the related equations on the board. Here are some possible student strategies:

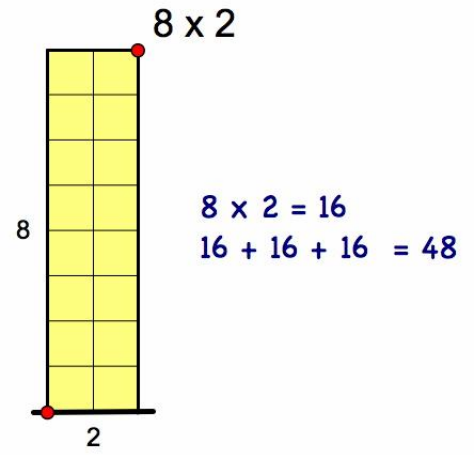
- *For the 8's facts, we split the array in half and added two 4's facts. To find 8 × 6, we used two 4 × 6 arrays.*



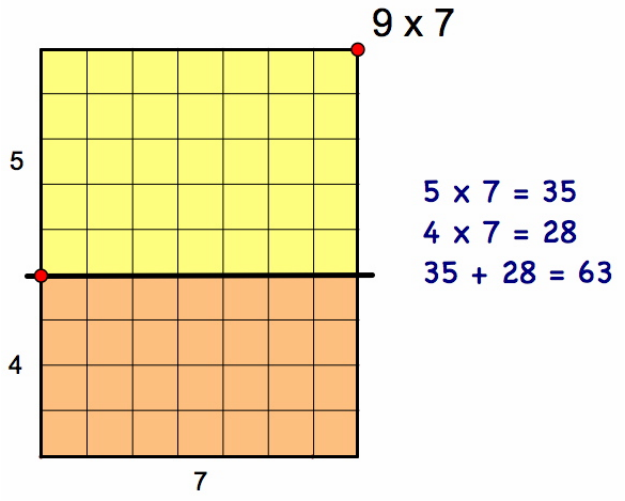
- *We split the 8's facts into 5's and 3's facts. The 5's and 3's facts are easier for us than the 4's facts. So, for the 8 × 6 array, we split it into a 5 × 6 array and a 3 × 6 array.*



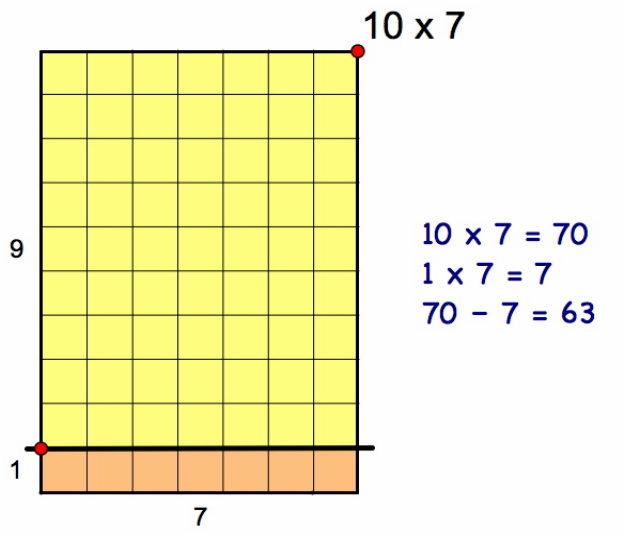
- *We saw that the 8 × 6 array was made up of three 8 × 2 arrays. We know our 2's facts, so we just found 8 × 2 and added it three times.*



- *We didn't bother finding 8 × 6 because we had already found 6 × 8. They are the same facts. The rows and columns of the arrays are switched. One has 8 rows and 6 columns. The other has 6 rows and 8 columns.*
- *We used the 5's facts and 4's facts to find the 9's facts. For example, we split a 9 × 7 array into 5 × 7 and 4 × 7 arrays.*



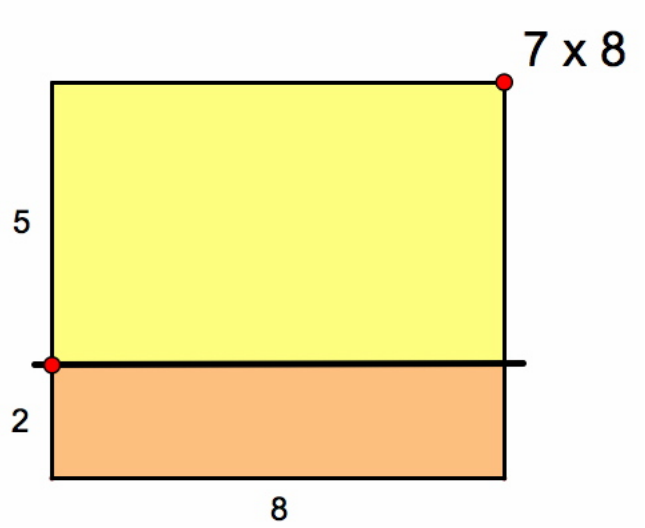
- *For 9's facts, we made up a subtracting strategy. We made a 10's array and subtracted a 1's array. For example, for 9 × 7, we made a 10 × 7 array and subtracted the 1 × 7 array.*



Allow sufficient time to explore all student-developed strategies. Seeing the different strategies will help students understand that there are many ways to compose and decompose numbers that are equally correct. Plus, they may see a new strategy that helps them memorize multiplication facts more easily.

Arrays Without Grid Lines:

In order for the array model of multiplication to be truly useful, students should be able to draw their own arrays without using Sketchpad. Drawing grid lines, of course, can be difficult. On page "Grid," show a problem like 7 x 8, but press *Hide Grid Lines* so that only the outline of the array appears.



Ask students if this picture of the array is sufficient for them to solve the problem. Then, drag the array control point to create other multiplication problems and confirm that grid lines are not necessary to interpret and use the arrays.

Finally, give students a multiplication problem and ask them to sketch on paper an array that they could use to solve the problem.

Related Activities:

- *Fill a Box with Chocolate—Skip Counting Techniques*
- *Bug Multiplication—Multiplication as Scaling*
- *Construct a Building—Multiplication Array Model*
- *Bunny Times*

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