

Notes for the Teacher

Students control the jumps of a rabbit along a number line. The rabbit can make jumps of two different integer distances, and these distances can be changed by you or your students. The rabbit might, for example, be limited to jumps of 1 and 2, or it might be able to take jumps of 10 and -3 . Don't be fooled by the cuteness of the rabbit: The range of questions that can be explored with this activity run from very simple to quite challenging!

Objectives:

Note: The Sketchpad model that accompanies this activity can be used in a variety of ways. The objectives below compile the learning outcomes across the suite of ideas presented in these notes. Depending on the grade you teach, you should touch upon only those objectives that are developmentally appropriate for your class.

- Students will use a number line model to explore properties of positive and negative numbers.
- Students will use a number line model to explore properties of even and odd numbers.
- Students will use a number line to explore and create number patterns.
- Students will use a number line model to explore tenths.

Common Core Mathematical Practices: (1) Make sense of problems and persevere in solving them; (2) Reason abstractly and quantitatively; (3) Construct viable arguments and critique the reasoning of others; (5) Use appropriate tools strategically; (7) Look for and make use of structure; (8) Look for and express regularity in repeated reasoning.

Common Core State Content Standards: K.CC2, 4; K.OA1; 1.OA5; 2.OA3; 2.NBT2; 4.OA5; 4.NF6; 5.NBT7; 6.NS5, 6a

Grade Range: Grades K–4

Introduce:

Open **Jump Along Games--Taking Trips on the Number Line.gsp** and go to page “Integer Puzzles.” You'll see a rabbit sitting at 0 on a number line. To change the scaling of the number line, drag any of the numbers sitting on the number line toward or away from 0 using the **Arrow** tool.

Only a few tick marks to the left of zero are visible. You can drag the entire number line with the **Arrow** tool to view the negative numbers (You'll need to press *Erase Traces* after dragging the number line.)

There are two *Jump By* values in the sketch, one of which is equal to 1, the other is equal to 2. By pressing the associated *Jump!* buttons, you'll see the rabbit jump by either 1 or 2. The rabbit leaves behind a trail of its jumps in either red or blue.

To make the rabbit jump from 0 to 5, you can jump by 1 five times, you can jump by 1 once and jump by 2 twice, or you can jump by 1 three times and jump by 2 once.

You can change the value of either *Jump By* to a different integer just by typing a new value into the edit box. These values can be either positive or negative. Negative jumps are represented by arcs below the number line.

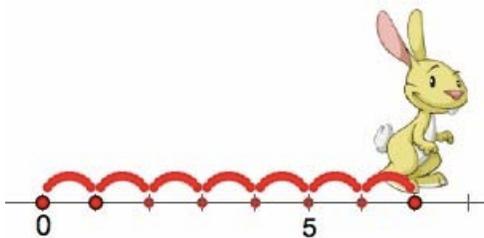
Pressing *Move Rabbit to 0* returns the rabbit to 0 and erases all traces of the rabbit's path. Pressing *Erase Traces* erases the trail of the rabbit but leaves the rabbit in place.

Below are some ideas of how you might use this model with students.

Jumping by 1

Start by setting the red *Jump By* value to 1 and the blue *Jump By* value to 0. Ask students to take several jumps of 1. Do they think it is possible for the rabbit to reach 10? 100? 1,000? Because the rabbit jumps by 1, it is possible to reach all of these destinations, although reaching 1,000 would take 1,000 jumps!

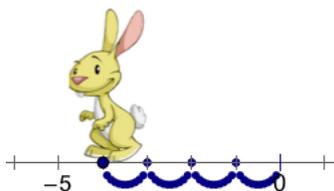
Ask students to describe all the locations that the rabbit can reach on the number line. The rabbit can jump to all the positive integers (1, 2, 3, 4, ...), but not to any locations to the left of zero (the negative integers).



Jumping by -1

Start by setting one *Jump By* value to -1 and the other *Jump By* value to 0. Ask students to take several jumps of -1 . In which direction does the rabbit hop? (to the left). How many jumps does it take the rabbit to reach -5 ? (5 jumps) -12 ? (12 jumps). Is it possible for the rabbit to reach -100 ? $-1,000$? (Yes, although $-1,000$ would take 1,000 jumps!

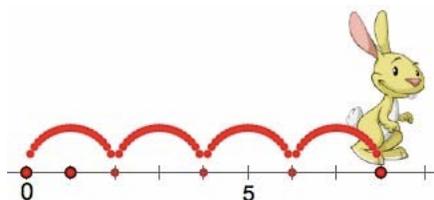
Ask students to describe all the locations that the rabbit can reach on the number line. The rabbit can jump to all the negative integers ($-1, -2, -3, -4, \dots$), but not to any locations to the right of zero (the positive integers).



Jumping to the Even Numbers

Start by setting one *Jump By* value to 2 and the other *Jump By* value to 0. Ask students to take several jumps of 2. Can the rabbit reach 10? Do students think it is possible for the rabbit to reach 100? 1,000? How about 13 or 95? The rabbit can only land on even numbers—those numbers that are multiples of 2.

Ask students how to predict how many jumps it takes for the rabbit to go from 0 to 12 (6 jumps). Then ask students to predict how many jumps it takes for the rabbit to go from 0 to 20 (10 jumps), 0 to 50 (25 jumps), and 0 to 100 (50 jumps). In general, it takes $\frac{n}{2}$ jumps for the rabbit to hop from 0 to n .

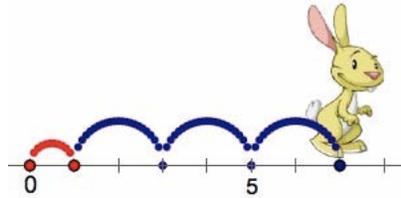


Jumping to the Odd Numbers

Start by setting one *Jump By* value to 1 and the other *Jump By* value to 2. Say, “Can someone name the first five odd numbers?” (1, 3, 5, 7, 9) Continue, “I’d like the rabbit to hop from one odd number to the next without missing any odd numbers in between. How can we do that?” (Jump from 0 to 1 and then jump by 2’s.)

Return the rabbit to 0. Ask, “I’d like the rabbit to jump from 0 to 15 in as few jumps as possible. How can we do that?” (Take as many jumps of 2 as possible (0, 2, 4, 6, 8, 10, 12, 14) as then take one final jump of 1.) Ask, “How can the rabbit jump from 0 to 101 in as few jumps as possible?” (Take jumps of 2 from 0 to 100 and then take one final jump of 1.) Finally, ask students to describe a general method for reaching any location along the number line in as few jumps as possible (If the location represents an even number, take jumps of 2. If the location represents an odd number, take as many jumps of 2 as possible and then a final jump of 1.)

As a challenge, ask students how many jumps it takes to reach 15 or 51 or 101. In general, to reach an odd number n , it takes $\frac{n-1}{2} + 1$ jumps.

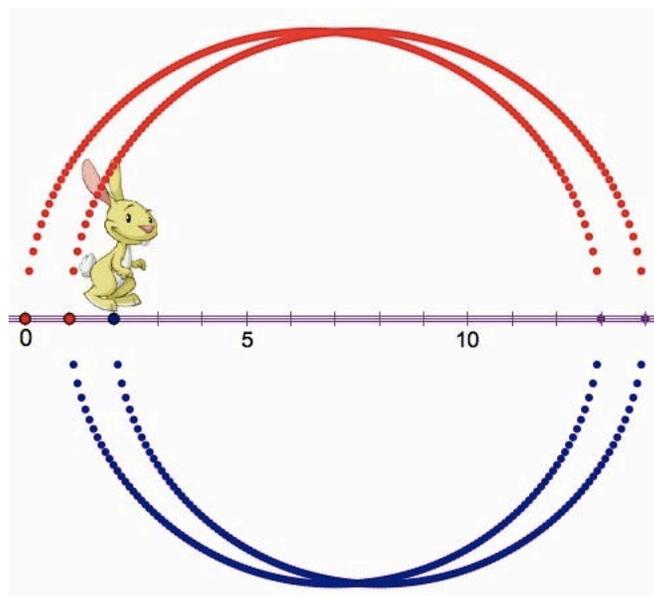


Jumping Back and Forth

Start by setting one *Jump By* value to 13 and the other *Jump By* value to -12 . Ask, “How can the rabbit jump to 1?” (The rabbit can jump from 0 to 13 and then from 13 to 1 by jumping by -12 .)

With the rabbit sitting at 1, ask the students how the rabbit can jump from 1 to 2. Again, the same strategy works: First, jump by 13 to go from 1 to 14, and then jump by -12 to go from 14 to 2. Continue this pattern for a while, asking students to jump from 2 to 3 to 4 and so on, noting the pattern formed by the rabbit repeatedly jumping forward by 13 and then backward by -12 .

Give students a related challenge: Set one *Jump By* value to 7 and the other *Jump By* value to -3 . Ask, “How can the rabbit jump to 1?” (The rabbit can jump from 0 to 7 and then take two jumps of -3). With the rabbit sitting at 1, ask the students how the rabbit can jump from 1 to 2. Again, the same strategy works: First jump by 7 to go from 1 to 8, and then jump by -3 twice to go from 8 back to 2.

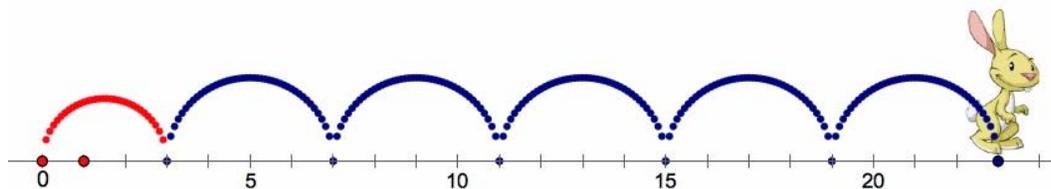


Following a Trail

Start by writing the following list of numbers on the board:

3, 7, 11, 15, 19, 23

Explain, “I’d like the rabbit to visit the numbers in this list in order. So I’d like the rabbit to travel from 0 to 3 and then from 3 to 7, from 7 to 11, and so on. You change the values of the two *Jump By* numbers. What should we try?” By setting one *Jump By* value to 3 and the other *Jump By* value to 4, students can follow the trail of numbers with the rabbit.



Give students some other number trails to try:

5, -1, -7, -13, -19, -25 (start at 5 and jump by -6)

-2, -5, -8, -11, -14, -17 (start at -2 and jump by -3)

Jumping to as Many Numbers as Possible

Start by setting one *Jump By* value to 3 and the other *Jump By* value to 5. Ask students whether they think the rabbit can visit every location on the number line. Students might explain that the rabbit cannot visit any destinations to the left of 0 because both *Jump By* values are greater than 0. Other students may note that the rabbit cannot visit 1 or 2.

Ask, “I wonder whether there are lots of positive integers that the rabbit cannot reach? I’d like you to see whether the rabbit can reach 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, and so on. Keep track of which numbers the rabbit can reach and which numbers it cannot reach.”

Give students plenty of time to explore. The result is surprising: While it is not possible to reach 4 or 7, it is possible to reach every number greater than or equal to 8! Why is this so?

For starters, note that it is possible to reach 8, 9, 10, 11, and 12:

$$8 = 5 + 3$$

$$9 = 3 + 3 + 3$$

$$10 = 5 + 5$$

$$11 = 3 + 3 + 5$$

$$12 = 3 + 3 + 3 + 3$$

Because it's possible to reach 8, 9, 10, 11, and 12, it is also possible to reach 13, 14, 15, 16, and 17—just add 5 to each value:

$$13 = 8 + 5$$

$$14 = 9 + 5$$

$$15 = 10 + 5$$

$$16 = 11 + 5$$

$$17 = 12 + 5$$

This pattern continues. To reach the next five numbers—18, 19, 20, 21, and 22—just add 5 to 13, 14, 15, 16, and 17.

For Your Information: If the *Jump By* values are a and b , where a and b share only 1 as a factor, then the rabbit can reach any number larger than $a \times b - (a + b)$.

Jumping by Tenths

Open the sketch and go to page “Tenths Puzzles.” On this number line, you can enter “Jump By” values in tenths and makes jumps less than 1. (You can drag one of the axis tick numbers right or left to change the scale. If you do this, be sure to move the rabbit to zero before or after changing the scale.)

Related Activities:

- *Number Navigation—Adding on an Open Number Line*
- *99 Chart—Mental Math Strategies*

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