Notes for the Teacher

Students add and subtract fractions with like denominators on a number line to reach a variety of target numbers. Students must build the fractions using only a specific list of values as numerators and denominators. Can students find more than one way to reach each target number by adding and subtracting different fractions? Students develop strategies for finding multiple solutions.

Objectives:

- Students will add and subtract fractions with like denominators.
- Students will develop strategies for adding and subtracting fractions to reach a given target using given numbers as choices for numerators and denominators.

Common Core Mathematical Practices: (1) Make sense of problems and persevere in solving them; (2) Reason abstractly and quantitatively; (3) Construct viable arguments and critique the reasoning of others; (5) Use appropriate tools strategically; (7) Look for and make use of structure.

Common Core State Content Standards: 4.NF3

Grade Range: Grades 4–5

Introduce:

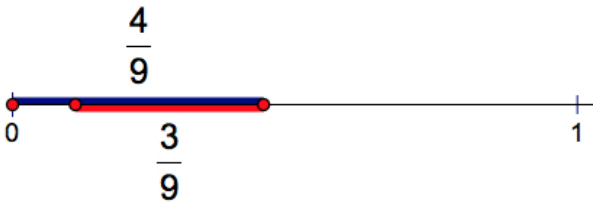
Use a projector to show sketch page "Game A" and distribute the worksheet. Explain, "I want to reach $\frac{1}{9}$ on the number line. I can make fractions using two tools: **Add Fraction** and **Subtract Fraction** (show the tools). Let's start by adding $\frac{4}{9}$ to 0."

Follow worksheet step 1, choosing **Add Fraction** to demonstrate how to add $\frac{4}{9}$ to 0.

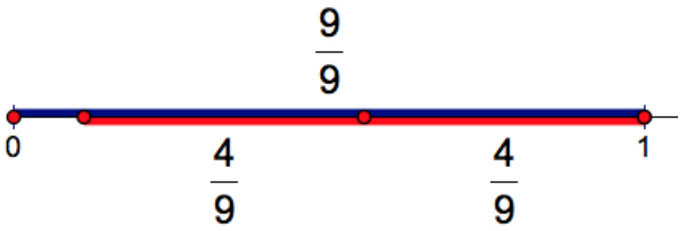
Notice that a blue line segment of length $\frac{4}{9}$ appears after you've clicked the number 4 and clicked the number 9. You can then move the segment and click to place it so that its left endpoint coincides with 0 on the number line.

Continue, "Now that we're at $\frac{4}{9}$, how can we reach our goal of $\frac{1}{9}$?" Students will likely suggest subtracting $\frac{3}{9}$ from $\frac{4}{9}$. Show how to subtract $\frac{3}{9}$ by choosing **Subtract**

Fraction. A red line segment of length $\frac{3}{9}$ appears after you've clicked the number 3 and clicked the number 9. You can then move the segment and click to place it so that its right endpoint coincides with the right endpoint of the previously placed $\frac{4}{9}$ line segment. Ask students to name the location of the left endpoint of the red line segment and explain their reasoning. The point is at $\frac{1}{9}$ because $\frac{4}{9} - \frac{3}{9} = \frac{1}{9}$. Help students make the connection between the fraction number line model and its corresponding number sentence. Check the answer by using the **Arrow** tool to press *Show* $\frac{1}{9}$.



Now ask, "Is there another way we can reach $\frac{1}{9}$ using the numbers 3, 4, and 9?" Explain that students can add or subtract two or more fractions. If students are stuck, suggest that they start by adding 1 to 0. They can create a fraction equal to 1 by clicking any of the three numbers 3, 4, or 9 twice in succession with the **Add Fraction** tool to make the fraction $\frac{3}{3}$, $\frac{4}{4}$, or $\frac{9}{9}$. Then ask students what to subtract from 1 to reach $\frac{1}{9}$. They can reach $\frac{1}{9}$ by subtracting $\frac{8}{9}$, but since there is no 8 available, they can subtract $\frac{4}{9}$ twice. Thus a second solution is $\frac{9}{9} - \frac{4}{9} - \frac{4}{9} = \frac{1}{9}$. Demonstrate how to build this number sentence on the second number line.



Now go to page "Game B" and ask, "How can we reach $\frac{9}{10}$ using addition and subtraction and the numbers 1 and 10 as numerators and denominators of our

fractions? Is there more than one way?" Give students some thinking time, and then let volunteers share their solutions. Students may suggest they can reach $\frac{9}{10}$ simply by adding $\frac{1}{10}$ nine times: $\frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{9}{10}$. Other students may recognize that $\frac{1}{1}$ or $\frac{10}{10}$ is equal to 1 and by subtracting $\frac{1}{10}$ from 1, they get $\frac{9}{10}$: $\frac{10}{10} - \frac{1}{10} = \frac{9}{10}$ or $\frac{1}{1} - \frac{1}{10} = \frac{9}{10}$.

Encourage students to use both the addition and subtraction tool for each challenge. Taken together, they will yield the most efficient solutions. That is, they will produce number sentences with the fewest fractions and the fewest operations.

Explore:

Assign students to partners, and send them in pairs to the computers. Tell students that they will play similar games similar to A and B, adding and subtracting fractions to reach a target number. Ask students to start on page "Game C" and then continue through page "Game J." Encourage students to find more than one combination of fractions and operations to reach the target number. For Games G and H, students will need to think about equivalent fractions with denominators that equal one of the given numbers. Give hints as needed to help students think of different fraction combinations that work.

As you circulate, observe students as they work. In Games C through F, the denominator is provided in the list of numbers, but the numerator is not. The challenge is to find ways to add and subtract the given numbers to produce the numerator. In Game C, for example, students may note that no combination of 4s and 5s equals 6 if using only addition. But $(2 \times 5) - 4 = 6$, so $\frac{5}{13} + \frac{5}{13} - \frac{4}{13} = \frac{6}{13}$.

Tell students that they can choose **Edit | Undo Add Fraction** and **Edit | Undo Subtract Fraction** to delete fractions they don't want to keep. Make sure students record the number sentences and their explanations on the worksheet.

Discuss:

Call students together to discuss and summarize what they've learned. Review the different ways to reach each target number. Have students share the strategies they used. Focus on those strategies that used both the **Add Fraction** and the **Subtract Fraction** tools together, as these strategies yield the most efficient solutions. Here are some sample student replies:

- *In Games B, F, and J, we could reach the target by using 1 in the numerator and adding the same fraction to itself over and over again. For example, in Game J, we could reach* $5\frac{5}{6}$ *by adding* $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\ldots$ *It took a long time, though! We looked for a quicker way.*

- *In Game J, we realized that* $5\frac{5}{6}$ *is very close to 6, so first we built* $\frac{6}{1}$*. Then, we subtracted* $\frac{1}{6}$ *because* $5\frac{5}{6}$ *is just* $\frac{1}{6}$ *away from 6.*

- *We tried different ways of adding and subtracting the given numbers to reach the numerator of the target fraction. We then placed our numbers over a common denominator and built the fractions.*

- *It helped to remember to use a number over itself to make 1 and any number over 1 to make a whole number. For example, in Game D, we used* $\frac{7}{7}-\frac{2}{7}$ *to reach* $\frac{5}{7}$*, and in Game I, we use* $\frac{10}{1}-\frac{2}{1}$ *to reach 8.*

- *In Game G, we thought of* $\frac{3}{4}$ *as the equivalent fraction* $\frac{6}{8}$ *because there was an 8 in the list of numbers but not a 4.*

Use a projector to show sketch page "Make Your Own." Have students work in pairs, using paper and pencil, to create their own game similar to the ones they've just played. Tell students they should pick a target number and state which numbers are available to build fractions to reach that target. (As a default, the sketch pages list the numbers 1, 1, 1, and 1, but students can use any numbers they like, and they don't need to provide four choices.)

When students are done, send them in pairs back to the computers and have them create their own games by changing the values on the sketch pages. Students should use the **Arrow** tool to double-click a number, enter a new value, and click **OK.** To change the target number, students should the **Arrow** tool to double-click the text box, highlight a question mark, and type a number on the keyboard. Have pairs share their games with the class or with other student pairs.

Related Activities:

- *Fractions on a Number Line—Sums of One*

- *Time-Saver Games, Part One—Adding Fractions*
- *Time-Saver Games, Part Two—Adding Fractions*
- *Fractions on a Number Line—Adding with Unlike Denominators*
- *Measuring with Fractions—Fractions on a Number Line*

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Portions of this material are based upon work supported by the National Science Foundation under award number DRL-0918733. Any opinions, findings, and conclusions or recommendations expressed in this work are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.