Notes for the Teacher

This activity challenges students to uncover the value of a secret number by collecting and analyzing clues that narrow its range of possible values. The activity familiarizes students with inequality signs (> and <), introduces the use of *x* to represent an unknown value, and motivates them to develop strategies for finding the secret number in the fewest guesses.

Objectives:

- *Students will interpret inequality symbols (> and <) signs correctly.*
- *Students will be introduced to the use of x to represent an unknown value.*
- *Students will develop, compare, contrast, and execute a variety of search strategies for identifying a secret number.*
- *Students will develop a sense of the uniform distribution of numbers on the integer number line.*
- *Students will extend their understanding of the ordering of integers to the negative integers.*

Common Core Mathematical Practices: (1) Make sense of problems and persevere in solving them; (2) Reason abstractly and quantitatively; (3) Construct viable arguments and critique the reasoning of others; (5) Use appropriate tools strategically; (7) Look for and make use of structure.

Common Core State Content Standards: 2.NBT4; 2.MD6; 6.EE2; 6.NS5

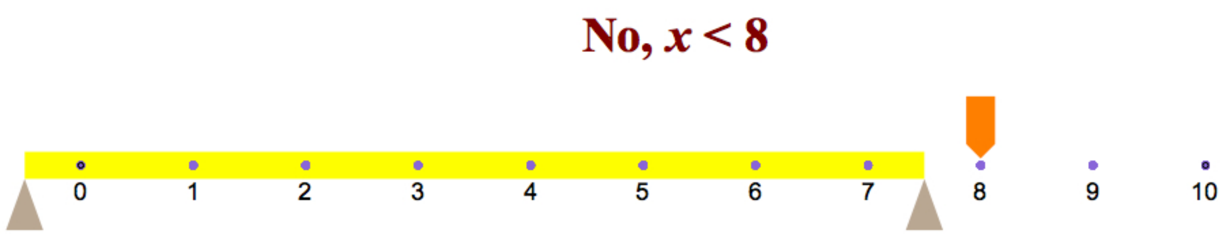
Grade Range: Grades 2–6

Introduce:

Use a projector to show sketch page 0–10, and distribute worksheet page 1. Explain, "The computer is thinking of a secret number and it's your job as a detective to use the clues you're given to find it. It's one of the numbers on this number line, but which one?" Call a student to the interactive whiteboard (if you have one) or to the computer to make the first guess by dragging the gold marker and then pressing the *Check my guess* button.

When each clue appears onscreen, have students write it down in row 1 of the first table of their worksheets. Ask several volunteers to interpret the clue, and review the meaning of the > and < signs. Discuss which suspects can be eliminated, and review what the words "suspect" and "eliminate" mean in this context, if necessary. Then have students cross out on their worksheets the suspects that they can eliminate, and have them write in the Suspects column the range of remaining suspects. Once students finish marking their worksheets, tell them, “To keep track of the clues on the sketch, we can drag the triangles to show which suspects have been eliminated.” Have the student drag the appropriate triangle to eliminate the same numbers that students crossed out on their worksheets.

In the example below, a student has dragged the gold marker to 8 as her first guess. Sketchpad tells her that the secret number is less than 8. The student then drags the rightmost triangle to eliminate suspects that are 8 or larger. The numbers highlighted in yellow—0 through 7—remain as suspects.



Call up a second volunteer to make the next guess by moving the gold marker, pressing the *Check my guess* button, and using the clue to eliminate more suspects. Continue with more volunteers until the class finds the secret number. Once they find the secret number, have students write the result (e.g., “$x = 6$”) in the Clue column and “Case closed” in the Suspects column.

This first whole-class investigation may be too short if the first or second guess turns out to be correct. If this happens, play a second game as a class to give students plenty of opportunity to see how to operate the game, and how to record clues and use them to determine the remaining suspects for each guess on their worksheets.

Avoid explaining strategies (such as using each guess to split the remaining interval of suspects in half) at this time. It's better for students to develop such strategies on their own through hands-on play, careful recording of the remaining suspects both on-screen and on paper, and discussion with their partner. If students offer strategies, respond by saying something like, “That’s an interesting strategy. You should try it when you work in pairs, and let us know how it works out.”

Explore:

Assign students to partners, send them in pairs to the computers, and ask them to play and record two games on page “0–10.” Tell them that partners should alternate turns, using the mouse to drag the gold marker, press the button, and drag the triangles. Then direct them to go to page “0–25” and to play at least two games at that level, recording their results on page 2 of the worksheet. (Students probably don't need to write each clue for the next levels.) Pass out page 3 of the worksheet and challenge students to earn the Level 3 Mathematician or Master Mathematician rank.

As you circulate, make sure students record the clues and the remaining suspects after each guess. (They can skip this once they're fluently using the > and < signs and using the triangles to keep track of remaining suspects.)

Some students may begin by testing integers in sequence (0, 1, 2, etc.) and progress to testing every multiple of 2 or of 5. Others may begin by testing their favorite "friendly" numbers (such as 10, 20, 30, ...). These strategies are reasonable for problems with relatively small intervals of integers, but become tedious for relatively large intervals of values. Page 3 of the worksheet challenges students to find five secret numbers from 0 to 200, using less than 30 clues altogether. This challenge is intended to motivate students to analyze and improve their strategy.

Be aware that the value of the most efficient strategy (halving the remaining interval) likely won't make sense to most students until they've tried out other strategies and found them wanting. It's important for students to own the process of developing and evaluating their own strategies in consultation with their partners. Don't rush things by describing a strategy students aren't ready for.

Use the "Make Your Own" page to challenge students to find a secret number between 200 and 210, or between 600 and 700. Encourage them to play around; it may be fun to choose a number line from 21350 to 21400. You can also use the "Make Your Own" page to suggest to students that they try the game with numbers to the left of 0. Practicing this game on intervals like [–10, 0] or [–25, 25] extends students' understanding of order and of the meaning of "less than" and "greater than" to negative integers.

Use the "Two Players" page so that pairs of students can compete against each other at the same time. Two students, one playing on the green number line, the other on the blue number line, should alternate taking guesses. Whoever can find his or her own secret number first wins!

Discuss:

Call students together with plenty of time for discussion and summarizing. Ask them what they've learned, and how they've figured out ways of solving the mystery with a small number of clues. (These explanations are mostly for the benefit of the students doing the explaining; students who haven't started using such strategies while playing the game are better served by playing the game again at a later time.)

Students may be amazed to realize that even though the 0-200 number line has twice as many numbers as the 0-100 number line, they at most need just one extra guess to find the secret number. A student might explain:

> *Solving the 0–200 number is easy. I just place the gold pointer at 100 to learn whether the secret number is less than or more than 100. Either way, I've just eliminated half the numbers. Now it's just like looking for the secret number on the 0–100 number line.*

Ask questions like "If you have only ten clues, what's the biggest number line on which you can be sure of finding the secret number?" "If your strategy is to first check the tens, by guessing 10, 20, and so forth, what's the largest number of guesses you might need on a number line that goes to 200?"

Related Activities:

- *Zooming Integers—Hundreds, Thousands, and Beyond*

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