Notes for the Teacher

Students investigate factor patterns in an interactive array. The array displays the numbers from 1 to *n* in color-coded form, so that factors of *n* are shaded blue while the remaining numbers are shaded gold. Students can easily change the value of *n* and observe the resulting pattern of blue and gold-shaded numbers to uncover factor patterns that otherwise would be difficult to spot.

Objectives:

- Students look for numerical and visual patterns in an interactive array that displays the factors of any number they choose.
- Students make and test conjectures about factors by using an interactive array.
- Students examine the properties of prime numbers in the course of reasoning about the factor patterns in their array.

Common Core Mathematical Practices: (1) Make sense of problems and persevere in solving them; (2) Reason abstractly and quantitatively; (3) Construct viable arguments and critique the reasoning of others; (5) Use appropriate tools strategically; (7) Look for and make use of structure.

Common Core State Content Standards: 3.OA5, 6, 7, 9; 4.OA4, 5; 6.NS4

Grade Range: Grades 3–5

Introduce:

Open **Factors in Blue and Gold--Explore Patterns of Factors.gsp.** Use a projector to show the "Array" page. Ask students to describe what they see. Students will likely say that the sketch shows the numbers from 1 to 32 arranged in a rectangular pattern (an *array*) with 8 numbers in each row. The majority of the numbers are shaded gold, but some are shaded blue. Students may recognize that those numbers shaded blue are the factors of 32. Ask students if there are any factors of 32 that are not shaded blue (There are not.)

Drag the red point to change the array so that the total number of circles is less than 32. Again, ask students to describe what they see. For every array, those numbers shaded in blue are the factors of the largest number in the array.

Keep the introduction to the model brief. It's best that students approach this model with as few prefacing comments as possible.

Explore:

Distribute the worksheet and review the directions. Assign students to partners and send them in pairs to the computers. Have students open **Factors in Blue and Gold--Patterns of Factors.gsp** and go to the "Array" Page. For students who are having difficulty deciding what to explore, offer the following suggestions:

- *Is there any location in the array that's always shaded blue? Why?*
- *How can we predict when the number 2 will be shaded blue?*
- *Show me all the arrays that have 24 circles.*
- *Show me all the arrays that have 23 circles.*
- *Do some portions of the array contain more blue circles than other portions? Can you explain why that might be?*
- *Can you make an array that has just one number shaded blue?*
- *What arrays have just two numbers shaded blue?*
- *Can you make arrays in which every number in the bottom row is shaded blue?*

Discuss:

Call students together to discuss the various observations they made about the factor patterns. Be sure that students also share any unanswered questions or hypotheses they might have. Here are some possible student responses:

- *The number 1 is always shaded blue.*
- *The number in the bottom-right corner of the array is always shaded blue.*
- *Our array has 11 rows, and I see that the number 11 is shaded blue. In general, if we have n rows in our grid, then the number n will be shaded blue.*
- *If the array has a prime number of circles, then there are only two ways to display it—either as a single row of circles or a single column of circles.*
- *We dragged the red point so that the numbers from 1 to 20 all appeared in a single row. We wanted to find other ways to display those 20 circles in the array. The numbers in blue—1, 2, 4, 5, 10, and 20—gave us a big hint. Since 2 is a factor of 20, we can make a 2 x 10 array. Similarly, we can make a 4 x 5, a 5 x 4, and a 10 x 2 array.*

- *We wanted to make an array to show the factors of 61. Of course, we could make a 1 x 61 array or a 61 x 1 array, but those wouldn't fit on the screen. We tried to make other arrays to show 61 but couldn't. We think that 61 is prime.*
- *If the array has at least 2 columns and 2 rows, then the number it represents isn't prime.*
- *When our array contains an even number of rows, the rows in the upper half of the array are filled entirely with gold circles. Only the number we're factoring is shaded blue. Why is that?*
- *Just two numbers are shaded blue when there are an odd number of circles in the array.*
- *If the number of rows and columns in the array are equal and prime, the array will contain exactly three numbers shaded blue.*
- *We found a way to create arrays with exactly four numbers shaded blue. Drag the red point to form a single row of numbers. Make sure the largest number in the row is prime. Then drag the red point straight up to create a prime number of columns. That does the trick.*
- *We can pair every number that is shaded blue with a partner. For example, the factors of 18 are 1, 2, 3, 6, 9, and 18. Let's pair 1 and 18, 2 and 9, and 3 and 6 together. In each pair, the product of the numbers is 18.*
- *What happens when we pair the factors of 25? Its factors are 1, 5, and 25. We can pair 1 and 25 together, but can we pair 5 with itself?*
- *In most of our arrays, there is an even number of circles shaded blue. But in some cases, the number of blue circles is odd. Is there a way to predict whether there will be an even or odd number of blue circles?*
- *If the number of columns is even, the number 2 is always shaded blue. If the number of columns is odd, the number 2 alternates between gold and blue as I drag the red point straight up to add more rows.*
- *We created a game. We scrolled our sketch window so that you can only see the bottom row of circles. Your challenge is to make an educated guess as to the total number of circles in the array.*

Related Activities:

- *Dancing Factors—Find Factors of a Number*
- *Dynamic Number Grids—Multiples and Patterns*

- *Dynamic Number Grids—Clues You Can Use*
- *Boxed Counters—Find Factors with an Array Model*

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Portions of this material are based upon work supported by the National Science Foundation under award number DRL-0918733. Any opinions, findings, and conclusions or recommendations expressed in this work are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.