

Notes for the Teacher

Students investigate the concepts of multiples, common multiples, and least common multiples by using an interactive grid. The grid displays multiples of any two numbers simultaneously, showing multiples of one number in orange, the other number in blue, and common multiples in an orange-blue combination. By experimenting with a variety of number pairs and different grid dimensions, students will be able to identify least common multiples and analyze visual and numerical patterns associated with common multiples.

Objectives:

- Students examine patterns relating to multiples, common multiples, and least common multiples in a interactive number grid.
- Students make and test conjectures about multiples, common multiples, and least common multiples.
- Students shade cells in an interactive number grid to display multiples of any number.
- Students shade multiples of any two numbers to determine their common multiples and least common multiple.

Common Core Mathematical Practices: (1) Make sense of problems and persevere in solving them; (2) Reason abstractly and quantitatively; (3) Construct viable arguments and critique the reasoning of others; (5) Use appropriate tools strategically; (7) Look for and make use of structure.

Common Core State Standards: 3.OA5, 6, 7, 9; 4.OA4, 5; 6.NS4

Grade Range: Grades 3–6

Using the Sketch: When you first open the Sketchpad model, you will see a grid of numbers from 1 to 120. Pressing *Show Orange Multiples* displays the multiples of *count by (orange)* as they appear one-by-one until all multiples are shaded orange. Pressing *Clear* restores the grid to its empty state. Pressing *Show Both Multiples* reveals the multiples of both *count by (orange)* and *count by (blue)* as they appear one-by-one until all multiples are shaded. Those numbers that are multiples of both *count by* values are shaded half orange, half blue.

Changing the values of *count by (orange)* and *count by (blue)* dynamically updates the grid to show multiples and common multiples of other numbers.

The grid's dimensions depend on two parameters, *columns* and *rows*. Changing these values dynamically updates the layout of the grid as well as its range of numbers.

Introduce:

Begin the activity as a whole class. Ask students to make a list of the first ten multiples of 3 and then a list of the first ten multiples of 4. Ask students to identify the least common multiple of 3 and 4 as well as those numbers on the lists that are common multiples of both numbers. Reinforce (or introduce) the vocabulary terms *multiple*, *common multiple*, and *least common multiple*.

Explore:

Distribute the worksheet and some orange- and blue-colored markers. Ask students, in pairs or small groups, to complete questions 1 and 2. Students will have to decide how to color the squares that represent common multiples. When students are done, ask them whether some grids were easier to color than others. Students might note that it was easy to color the multiples of 3 in the grid with three columns because all the multiples appeared in a single column.

Introduce:

Open **Dynamic Number Grids--Multiples and Patterns.gsp** and go to page "Grid." Press *Show Orange Multiples* and ask students to watch as the multiples of 3 are shaded orange. Demonstrate how to change the number of columns in the grid: Click in the *column* edit box and type a new value, or click the *column* label and press the + or – key on your keyboard to increment or decrement the value by 1. Slowly change the number of columns from 10 to 9 to 8 to 7 to 6 and ask students to write down which values make it easiest for them to see a pattern. Students may note that when *columns* is a multiple of 3, the orange multiples line up in columns.

Press *Clear* to return to an empty grid. Press *Show Blue Multiples* to reveal the multiples of 4, all shaded blue. As before, experiment with changing the number of columns in the grid to display different patterns.

Press *Clear* again. Ask students what they think will happen when you press *Show Both Multiples*. Some students might think that the common multiples of 3 and 4 will be shaded a color other than orange and blue; other students might think the squares will be shaded half orange, half blue. Press *Show Both Multiples* to reveal what happens. Ask students how they can use the grid to identify the least common multiple of 3 and 4 as well as common multiples of 3 and 4.

Ask, "Can we change the number of columns so that *all the common multiples* are in the same column, and there are *no other numbers* in that column?" Solicit several opinions, and ask for reasoning, before testing proposed solutions.

Explore:

Divide students into pairs and invite students to work through worksheet questions 3–4.

Question 3 focuses on the grid patterns formed by common multiples. In particular, students discover that by setting the parameter *columns* equal to the least common multiple of two numbers, the right-hand column will contain *all the common multiples and no other numbers*. For some pairs of numbers, like 5 and 3, students will not be able to see every column of the grid when they change the value of *columns*. Encourage students to enlarge the sketch window and scroll to the right and up and down to see the remainder of the grid.

Possible Misconceptions: Students may generalize that the least common multiple of two numbers is equal to their product. Questions 2 and 3 give students examples where this is not the case. Challenge students to identify number pairs for which the least common multiple *is* the product of the two numbers. They might notice that when two numbers share no factors in common other than 1, their least common multiple is their product.

Discuss:

Bring the whole class back together and discuss the patterns they noticed in each question. Are students able to explain how to make the common multiples of two numbers appear in the same column?

For students who need more practice with common multiples and least common multiples, have them use the Dynamic Number Grids sketch on another day, giving them more problems like question 3 on the worksheet.

Worksheet Answers:

1. The least common multiple of 3 and 5 is 15.
2. Common multiples of 2 and 6 include 6, 12, 18, and 24. The smallest of these multiples, 6, is the least common multiple of 2 and 6.
- 3.

Count by (orange)	Count by (blue)	Common Multiples	Least Common Multiple	How Many Columns?
2	5	10, 20, 30	10	10
3	6	6, 12, 18	6	6
5	3	15, 30, 45	15	15

4	3	12, 24, 36	12	12
4	6	12, 24, 36	12	12
3	1	3, 6, 9	3	3
5	10	10, 20, 30	10	10
8	12	24, 48, 72	24	24

Related Activities:

- *Dancing Factors—Find Factors of a Number*
- *Factors in Blue and Gold—Explore Patterns of Factors*
- *Boxed Counters—Find Factors with an Array Model*
- *Dynamic Number Grids—Clues You Can Use*

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