Notes for the Teacher

Students develop their flexibility in thinking about multiples and common multiples through solving puzzle-like problems. Students are given information about the multiples and common multiples of two unknown numbers to use as clues in determining the identity of the numbers. An interactive grid serves as the unifying model for the problems, with students using it to check their work as well as to create puzzle challenges for each other.

Objectives:

- Students use a picture of part of a number grid to determine the number of columns in the grid.
- Students use a partial grid on which all multiples of an unknown number are shaded orange to form a conjecture about the value of the unknown number.
- Students use a partial grid with multiples of one number in orange and multiples of another in blue to form a conjecture about the values of the two numbers.
- Students verify their conjectures and determine whether they are unique.
- Students examine shaded cells in a grid without numbers to determine the skip-count interval between them, and fill in the cells with a possible sequence of numbers.

Common Core Mathematical Practices: (1) Make sense of problems and persevere in solving them; (2) Reason abstractly and quantitatively; (3) Construct viable arguments and critique the reasoning of others; (5) Use appropriate tools strategically; (7) Look for and make use of structure; (8) Look for and express regularity in repeated reasoning.

Common Core State Content Standards: 3.OA5, 6, 7, 9; 4.OA4, 5; 6.NS4

Grade Range: Grades 3–6

Prerequisite: Students should first complete the activity Dynamic Number Grids—Multiples and Patterns.

Using the Sketch: When you first open the Sketchpad model, you will see the multiples of two numbers, 2 and 3, simultaneously on the grid. The multiples of 2 are shaded orange and the multiples of 3 are shaded blue. Those numbers that are multiples of **both** 2 and 3 are shaded half orange, half blue. Changing the values of *count by (orange)* and *count by (blue)* dynamically updates the grid to show multiples and common multiples of other numbers.

The grid's dimensions depend on two parameters, *columns* and *rows*. Changing these values dynamically updates the layout of the grid as well as its range of numbers.

Introduce:

Open **Dynamic Number Grids--Clues You Can Use.gsp** and go to the "Grid" page. Review with students how the model works, showing the effects of changing the *count by* values as well as the value of *columns*.

Distribute the worksheet. The questions on the worksheet are challenging, so you'll want to discuss an example of each type of problem as a whole class.

Ask students to look at question 1a, and go to the "Make Your Own" page of the sketch. The question and the sketch display just a portion of the entire number grid. The values of *count by (orange)*, *count by (blue)*, and *columns* have all been hidden. Discuss with students how they can figure out these values using the information in the grid.

To determine the value of *columns*, students should figure out that the difference between any two adjacent numbers in a column is equal to the number of columns in the grid (referring back to page "Grid" may help). Thus *columns* = 9.

Students will need to reason backwards to think about what pairs of numbers can produce the shading in the grid. Some students may suggest that *count by (blue)* = 3, because 3 is a factor of both 12 and 30. They should note, however, that 3 is not shaded, so 3 cannot be the value of *count by (blue)*. Similar reasoning should lead students to eliminate 2 as a value for *count by (orange)*. Conclude that *count by (orange)* equals 4 and *count by (blue)* equals 6. Press *Show Values* to check.

Ask students to look at question 2a. The picture shows a grid with its numbers missing and some of its rows cut off. The challenge is to put the numbers back into the grid. At first, this may seem impossible! But there are clues that can help. There are seven columns in the grid, so the numbers in the right-most column are multiples of 7. The shaded squares are two apart, so *count by (blue)* = 2. Thus the right-hand shaded square in second fully visible row of the grid must be a multiple of both 7 and 2. Possible values for this square are 28, 42, 56, etc. (It can't be 14 because there are at least two rows above this row.)

Explore:

Divide students into pairs and invite students to work through worksheet questions 1 and 2.

For question 1, there is more than one possible answer to parts b–e. Encourage students to find and list as many numbers as possible that produce the given grids. For question 1e, students will discover that any value for *count by (orange)* and *count by (blue)* that

is larger than 75 will work. Ask students to find some values less than 75 that will work as well.

Discuss:

Bring students back together and go to the "Make Your Own" page of the sketch. Call a volunteer to the computer that's being projected, cover the projector's lens, and ask the volunteer to follow the directions in the sketch to create a new problem for her classmates to solve. Because some problems have more than one answer, students will discover that the answer intended by the volunteer does not necessarily match the valid answers suggested by the class.

Worksheet Answers:

1. a. *count by (orange)* = 4; *count by (blue)* = 6

   b. *count by (orange)* = 5 or 15; *count by (blue)* = 9, 18, or 36; *columns* = 7

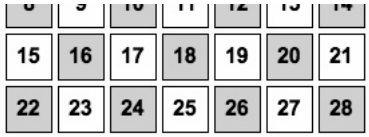
   c. *count by (orange)* = 2, 3, or 6; *count by (blue)* = 7, 14, 21, or 42; *columns* = 12

   d. *count by (orange)* = 49; *count by (blue)* = 15 or 45; *columns* = 5

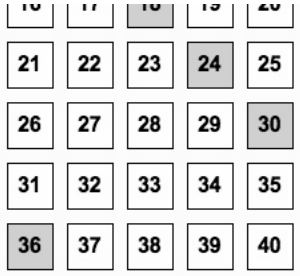
   e. An infinite number of values for *count by (orange)* and *count by (blue)* are possible, including any number larger than 75; *columns* = 13

2. One possible answer for each question is given below.

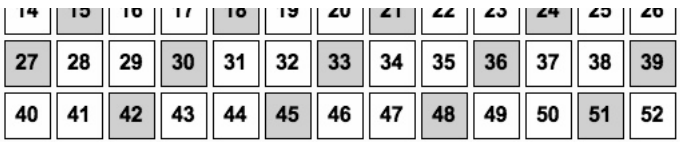
   a.



   b.



   c.



Related Activities:

- *Dancing Factors—Find Factors of a Number*

- *Factors in Blue and Gold—Explore Patterns of Factors*
- *Boxed Counters—Find Factors with an Array Model*
- *Dynamic Number Grids—Multiples and Patterns*

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