

# Dilation Challenges (long form) Name: \_\_\_\_\_

In this activity you will dilate a point and compare its motion to the motion of its dilated image.

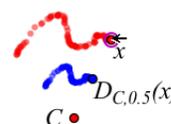
## DILATE A POINT

Begin by dilating a point and describing how the dilation behaves.

1. In a new sketch, construct an independent point  and label it  $x$  .
2. Drag the point  around in your sketch.  
Because you can drag this point anywhere in your sketch, and it doesn't depend on other objects, we say that point  $x$  is the *independent variable*.
3. Construct another point  in the center of the sketch. Label it  $C$  .
4. With point  $C$  selected, choose **Transform | Mark Center**.
5. Select point  $x$   and choose **Transform | Dilate**. The scale factor is  $1.0 / 2.0$ , which is equal to  $0.5$ . Click the Dilate button to create the dilated image.
6. Label the dilated point   $D[C,0.5](x)$ . Because point  $D_{C,0.5}(x)$  depends on point  $x$ , we call it the *dependent variable*.

You can read  $D_{C,0.5}(x)$  as “the dilation of  $x$  about  $C$  by one half.”

7. Change the color of the dependent point  $D_{C,0.5}(x)$  by selecting it  and choosing **Display | Color**.
- Q1** Drag  $x$  up. Which way does  $D_{C,0.5}(x)$  go? How fast does it go relative to point  $x$ ?  
Drag  $x$  to the left. Which way does  $D_{C,0.5}(x)$  go? How fast does it go?
8. Turn on tracing for points  $x$  and  $D_{C,0.5}(x)$ , by selecting them both  and choosing **Display | Trace Points**.
9. Click in empty space  to deselect both points. Then drag independent point  $x$  to trace out an interesting shape.
- Q2** Describe the traced shapes. How are they similar, and how are they different? Consider position, size, angle, and anything else you think of. On your paper include a drawing of your two traced shapes.
10. Erase the traces by choosing **Display | Erase Traces**, and then trace a new shape. Make this shape go through the center point.
- Q3** What happened when you went through the center? Describe these traced shapes, and include a drawing on your paper. Were there any fixed points? If so, where?



A location where  $x$  and  $D_{C,0.5}(x)$  come together is called a *fixed point* of the function.

- Q4** Click the **Information** tool  on point  $x$ , and then on  $D_{C,0.5}(x)$ . How do the resulting balloons describe each point?

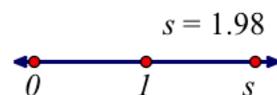
## USE A DIFFERENT SCALE FACTOR

- Choose **File | Document Options** and add a new page to your document.

In the dialog box, press the Add Page button. Choose **Blank Page** and then press OK.

- Choose the **Line** tool . While holding the Shift key, construct a horizontal line near the bottom of your sketch. Label the line's two points 0 and 1.
- Construct a point  on the line, between point 0 and point 1. Label it  $s$ .

- Select point  $s$  and choose **Measure | Value of Point**. Label the measurement  $s$ . Drag point  $s$  to see how measurement  $s$  behaves. Leave point  $s$  where its value is about 2. (Point  $s$  is a *scale slider*, because you can slide it to change the scale factor.)



- As you did on page 1, construct independent variable  $x$   and point  $C$ , and mark point  $C$  as the center for dilation.
- Select point  $x$   and choose **Transform | Dilate**. With the Dilate dialog box open, click measurement  $s$  as the scale factor. Then click the Dilate button.
- Label the dilated point   $D[C,s](x)$ . This is the *dependent variable*.

**Q5** Drag  $x$  up. Which way does  $D_{C,s}(x)$  go, and how fast? Drag  $x$  left. Which way does  $D_{C,s}(x)$  go, and how fast?

- Select both variables and turn on tracing. Select  $x$  only and drag it to make a shape.

**Q6** On your paper describe and draw the traced shapes. How are these shapes different from the shapes you made when you dilated by 0.5?

**Q7** Drag the value of  $s$  to about 0.3, erase the traces, and drag  $x$  again. How are these new shapes different from the previous shapes? How are they similar?

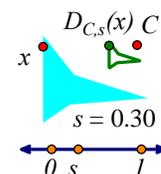
**Q8** Can you drag the two points together to find a *fixed point*? If so, where is it?

## RESTRICT THE DOMAIN

Now you'll merge point  $x$  to a polygon and observe the effect on  $D_{C,s}(x)$ .

- Construct an interesting polygon  with at least 5 vertices. To finish the polygon, click again on the first point.
- To hide the vertices of the polygon, select the vertices  (but not the polygon) and choose **Display | Hide Points**.
- Merge point  $x$  to the polygon by selecting  both the point and the polygon and choosing **Edit | Merge Point to Polygon**.

**Q9** Drag  $x$  , and describe the function's domain. In other words, where can you drag the independent variable?



A domain like this is called a *restricted domain*.

19. Select  $x$  , and choose **Edit | Action Button | Animation**. Then press OK.
  20. Erase the traces and press your new *Animate Point* button.
- Q10** How does the trace of  $D_{C,s}(x)$  (the *range*) compare to the restricted domain? What features of the domain and range are similar, and what features are different?

## A DILATION CHALLENGE

Now you'll look at some dilations and find the location of the invisible center and the unknown scale factor.

21. Open **Dilation Challenges.gsp** and try the Challenges.
- Q11** Describe in step-by-step detail how you can create a new dilation to exactly match any given dilation. How can you locate of the invisible center of dilation, and how can you find the scale factor? Describe any shortcuts you've invented to make it easier to find either of these features.

# Dilation Challenges (short form) Name: \_\_\_\_\_

In this activity you will dilate a point and compare the motion of the point to the motion of its dilated image.

## DILATE A POINT

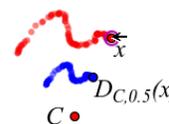
Begin by dilating a point and describing how the dilation behaves.

1. In a new sketch, dilate point  $x$  about center point  $C$  using the scale factor of  $1.0/2.0$ . Label the dilated point  $D[C,0.5](x)$ , and change its color. Because point  $D_{C,0.5}(x)$  depends on point  $x$ , we call it the *dependent variable*.

You can read  $D_{C,0.5}(x)$  as “the dilation of  $x$  about  $C$  by one half.”

**Q1** Drag  $x$  up. How does  $D_{C,0.5}(x)$  move? Drag  $x$  left. How does  $D_{C,0.5}(x)$  move?

2. Turn on tracing for points  $x$  and  $D_{C,0.5}(x)$ . Deselect both points and then drag independent point  $x$  to trace out an interesting shape.



**Q2** Describe the traced shapes. How are they similar, and how are they different? Consider position, size, angle, and anything else you think of. On your paper include a drawing of your two traced shapes.

3. Erase the traces and trace a new shape that goes through the center point.

**Q3** What happened when you went through the center of dilation? Describe these traced shapes, and include a drawing on your paper. Were there any fixed points? If so, where?

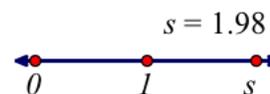
A location where  $x$  and  $D_{C,0.5}(x)$  come together is called a *fixed point* of the function.

**Q4** Click the **Information** tool on point  $x$ , and then on  $D_{C,0.5}(x)$ . How do the resulting balloons describe each point?

## USE A DIFFERENT SCALE FACTOR

4. On a new page of your document, construct a horizontal line near the bottom of your sketch. Label the line's two points 0 and 1.

5. Construct a point on the line, between point 0 and point 1. Measure the value of the point, and label both the point and its measurement  $s$ . Drag point  $s$  so that its value is about 2.



6. Construct independent variable  $x$  and center point  $C$ , and dilate  $x$  about center point  $C$ . Label the dilated point  $D[C,s](x)$ , and change its color. Point  $D_{C,s}(x)$  is the *dependent variable*.

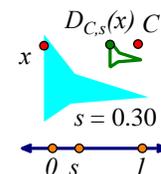
**Q5** Drag  $x$  up. Which way does  $D_{C,s}(x)$  go, and how fast? Drag  $x$  left. Which way does  $D_{C,s}(x)$  go, and how fast?

7. Turn on tracing for both variables, and then drag point  $x$  only to make a shape.

- Q6** On your paper describe and draw the traced shapes. How are these shapes different from the shapes you made when you dilated by 0.5?
- Q7** Drag the value of  $s$  to about 0.3, erase the traces, and drag  $x$  again. How are these new shapes different from the previous shapes? How are they similar?
- Q8** Can you drag the two points together to find a *fixed point*? If so, where is it?

## RESTRICT THE DOMAIN

Now you'll restrict point  $x$  to a polygon and observe the effect on  $D_{C,s}(x)$ .



8. Construct an interesting polygon with at least 5 vertices, hide the vertices, and merge the independent variable to the polygon.
- Q9** Drag  $x$  , and describe the function's domain. In other words, where can you drag the independent variable?
- A domain like this is called a *restricted domain*.
9. Erase the traces, and create an Animation button to animate the independent variable around the polygon. Press your new *Animate Point* button.
- Q10** How does the trace of  $D_{C,s}(x)$  (the *range*) compare to the restricted domain? What features of the domain and range are similar, and what features are different?

## A DILATION CHALLENGE

Now you'll look at some dilations and find the location of the invisible center and the unknown scale factor.

10. Open **Dilation Challenges.gsp** and try Challenges 1 through 6.
- Q11** Describe in step-by-step detail how you can create a new dilation to exactly match any given dilation. How can you locate of the invisible center of dilation, and how can you find the scale factor? Describe any shortcuts you've invented to make it easier to find either of these features.

## Dilation Challenges Answers    Name: \_\_\_\_\_

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- Q1** Drag  $x$  up. Which way does  $D_{C,0.5}(x)$  go? Drag  $x$  left. Which way does  $D_{C,0.5}(x)$  go?
- Q2** Describe the traced shapes. How are they similar, and how are they different? Consider position, size, angle, and anything else you think of. On your paper include a drawing of your two traced shapes.
- Q3** What happened when you went through the center of dilation? Describe these traced shapes, and include a drawing on your paper.
- Q4** Click the **Information** tool on point  $x$ , and then on  $D_{C,0.5}(x)$ . How do the resulting balloons describe each point?
- Q5** Drag  $x$  up. Which way does  $D_{C,s}(x)$  go, and how fast? Drag  $x$  left. Which way does  $D_{C,s}(x)$  go, and how fast?
- Q6** On your paper describe and draw the traced shapes. How are these shapes different from the shapes you made when you dilated by 0.5?

## Dilation Challenges Answers

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- Q7** Drag the value of  $s$  to about 0.3, erase the traces, and drag  $x$  again. How are these new shapes different from the previous shapes? How are they similar?
- Q8** Can you drag the two points together to find a *fixed point*? If so, where is it?
- Q9** Drag  $x$ , and describe the function's domain. In other words, where can you drag the independent variable?
- Q10** How does the trace of  $D_{C,s}(x)$  (the *range*) compare to the restricted domain? What features of the domain and range are similar, and what features are different?
- Q11** Describe in step-by-step detail how you can create a new dilation to exactly match any given dilation. How can you locate the invisible center of dilation, and how can you find the scale factor? Describe any shortcuts you've invented to make it easier to find either of these features.

# Dilation Challenges Exit Ticket

Name: \_\_\_\_\_

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1. Describe one important thing you learned today about the dilation function family.

2. Describe one thing about the dilation function family that seems confusing to you.