

## Notes for the Teacher

In this two-part activity, students perform dances based on several different function families they have explored (translation, reflection, rotation, and dilation). In the first part, teams of students practice and perform a dance, with one student in the role of the independent variable, another in the role of the dependent variable, and the remaining students as choreographers who coach dancers to follow the function rule. In the second part, students work in pairs to perform similar dances on the computer, with one student in the role of dependent variable, dragging a point to follow the function rule, and the other student acting as coach.

Through the two parts of this activity, students will gain a stronger sense of how variables behave, of their relative rate of change (covariation), and of the similar and congruent patterns made by a restricted domain and its corresponding range. Cognitive science suggests that students form abstract concepts (such as *variable* and *function*) through *conceptual metaphors* grounded in their experiences. This activity is designed to provide students with conceptual metaphors both for variables (as students either vary their physical location by dancing or vary a point's location by dragging) and for functions (as students follow the self-enforced rules of the physical dance or the Sketchpad-enforced rules of the virtual dance).

This is one of a series of Geometric Functions<sup>1</sup> activities in which students explore geometric transformations as functions. By using points as their independent and dependent variables, students can vary the independent variable and observe directly the behavior of the dependent variable. Students are encouraged to pay attention to the relative rate of change of the two variables and to other characteristics of the function (such as its fixed points). They trace the variables to record their locations over time (thus developing both *covariation* and *correspondence* views of a function), and they relate the shapes formed by the traces to their observations about relative rate of change and fixed points of the function. With this approach students directly manipulate variables to explore domain, range, composition, and inverse, making these concepts visible through dynamic images that reveal their fundamental aspects.

### **Objectives:**

In this activity students will:

- Work with a team to practice and then perform several function dances chosen from the reflection, rotation, dilation, and translation families.
- Dance the role of an independent variable either by moving freely or by moving along a restricted domain.

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<sup>1</sup> *Geometric Functions* (plural, capitalized) is used here to refer to this sequence of activities in which students explore geometric transformations as functions. A *geometric function* (lowercase) is used to refer to any transformation that takes a point to a point.

- Dance the role of a dependent variable, moving in coordination with the independent dancer to maintain the function rule.
- Coach the independent and dependent dancers to help them maintain the function rule.
- Drag a point to keep it in the role of dependent variable for an animated independent variable.
- Observe, analyze, and record the relative rate of change of the two point variables.
- Compare the shapes traced by an independent variable restricted to a domain and a corresponding dependent variable, distinguishing between similar shapes and congruent shapes.

**Common Core Mathematical Practices:** (1) Make sense of problems and persevere in solving them; (2) Reason abstractly and quantitatively; (3) Construct viable arguments and critique the reasoning of others; (4) Model with mathematics; (5) Use appropriate tools strategically; (6) Attend to precision; (7) Look for and make use of structure; (8) Look for and express regularity in repeated reasoning.

**Common Core State Content Standards:** 8.F.1,2; 8.G.1; F-IF.1,2,9; G-CO.2; G-SRT.1

**Grade Range:** Grades 7–11

**Prerequisites:**

Before undertaking this activity, students should have done one or the other of the following:

- Three of the four function challenge activities (*Reflection Challenges, Rotation Challenges, Dilation Challenges, Translation Challenges*), or
- *Family Relationships—Rotation, Dilation, and Translation Families.*

These prior activities are also recommended:

- *ID the Suspects—Identify Functions*
- *Family Resemblances—Identify Function Families*

**Instructional Strategies:**

This activity incorporates a number of instructional strategies designed to develop students' conceptual understanding of functions.

**High Cognitive Demand:** This activity provides tasks for which there is no established procedure for students to follow. The questions asked on the worksheet are at a fairly high level, and require experimentation, inquiry, and analysis to answer.

**Mathematical Habits of Mind, Reasoning and Sense Making:** Students put themselves in the role of mathematical objects, and are challenged to behave like independent variables (moving freely or restricted to a domain) and like dependent variables (moving to maintain a particular functional relationship to the independent variable).

**Inquiry:** The activity supports student inquiry, and the worksheet contains probing questions that require students to manipulate, observe, and analyze. Students answer questions such as: When I'm the dependent variable, what direction and how fast must I move when the independent variable moves? Do the domain and range exhibit similarity or congruence? The personalization of the variables in this activity makes the inquiry particularly compelling: instead of asking "Where should the variable go?" students ask "Where should I go?"

**Cooperative Learning:** Students work in teams during the physical dance part of the activity, and in pairs during the virtual dance. In both parts students play a variety of roles, both the lead roles (as variables) and the supporting roles (as choreographers). Expect students to work with purpose and focus in groups, to coach each other, and to discuss every part of the activity with their partners.

**Assessment:** You should engage in formative assessment by visiting and questioning student teams, not only observing the dancers but also encouraging and guiding the choreographers. Use the summary discussion to probe for students' conceptual understanding and misconceptions. The last page of the worksheet is an exit ticket.

**Differentiation:** This activity is designed to engage students with different levels of background knowledge and skills. Students choose what dances to do, they have the support of their team in the physical dance and of their partner in the virtual dance, and they can create their own virtual or physical dance in the Explore More section.

**Questioning and Discourse:** Most discourse will take place between team members during inquiry, so it's important to encourage team members to describe and explain to each other what they observe, and to strategize about how to make sure the function rule is followed. Use the summary discussion to focus students' thinking on the big ideas, and the role they play in both the physical dance and the virtual dance.

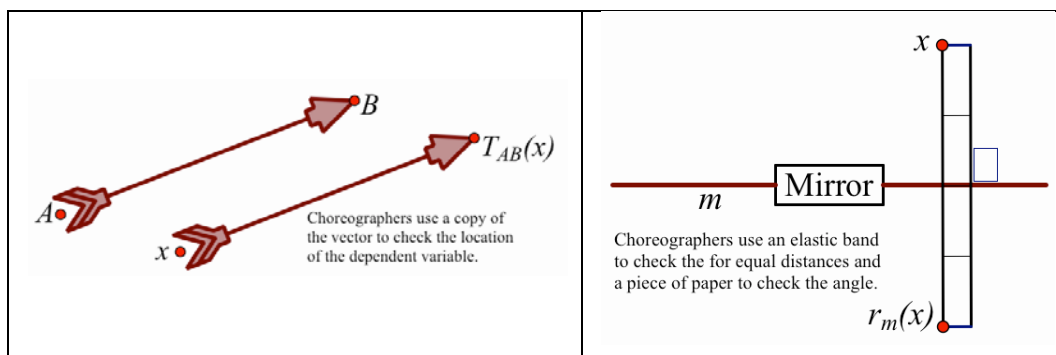
**Instructional Strategies:** By varying the independent variable, students are already investigating similarities and differences by comparing what features stay the same with what features change. This activity also makes strong use of multiple representations, of developing and testing hypotheses, and of feedback that doesn't depend on the teacher.

**Preparation:**

Performing the physical function dances will take a full class period (45 minutes). Performing the virtual dances will take less than a full period, and can be combined with discussion and review of students' experiences with the physical dances.

Prepare by printing enough copies of the worksheet (**Dance the Dependent Variable Worksheet.pdf**) for your class. The worksheet file contains the instructions as well as Function Dance Cards for students to fill out during the physical and virtual parts of the activity. Also print enough copies of the graphics (**Dance the Dependent Variable Graphics.pdf**) to make the necessary props for students to use. The suggested props are described here:

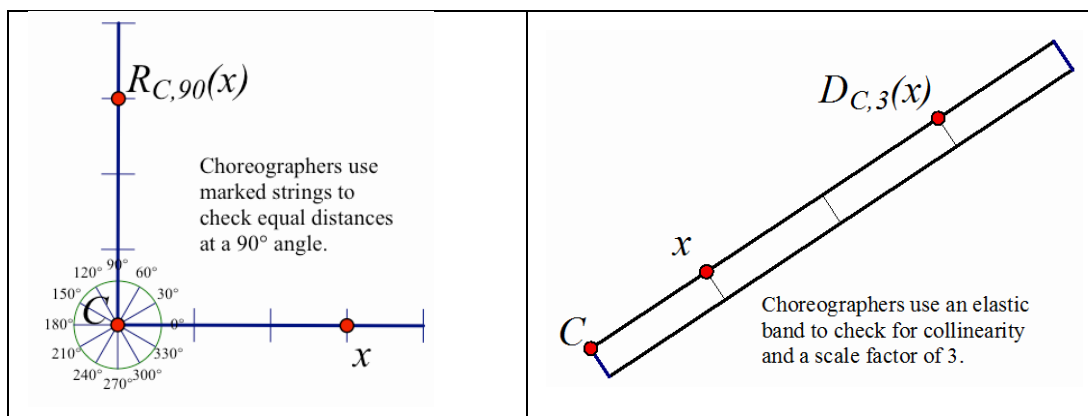
*Translation Dance:* Cut a piece of string or rope about 3 m in length and make it into a vector by stapling each end of the rope between two pieces of cardboard, and fastening two arrowhead printouts (from the Graphics pdf) to one piece of cardboard and two arrow tail printouts to the other piece of cardboard. During use, the vector can be shortened by forming a loop and tying a temporary knot in the loop. During the dance, choreographers will use a copy of the vector to check the distance and direction between the dancers.



*Reflection Dance:* Attach a “Mirror” printout to a piece of string or rope about 3 m long. You can use a tape measure, a folding rule, or a string to measure the distance of each dancer from the mirror. However, measurements are even easier with a 2 m length of flat elastic exercise band marked in fourths. (The lighter and more stretchy the elastic, the better. You can find 50-yard rolls online by searching for “extra light 50 yard exercise band.”) During the dance, two choreographers hold the mirror while two other choreographers periodically check the positions of the dancers by measuring distances or stretching a band to make sure the dancers are equally distant from the mirror and that the stretched band or tape measure makes a right angle with the mirror.

*Rotation Dance:* Attach the protractor printout to a piece of cardboard and punch a hole in the center. Insert two long pieces of string (about 3 m each) through the hole and fasten their ends to the bottom of the protractor with tape or staples. Use paint or a marker to put tick marks at regular intervals along both two strings. Finally, choreographers tape the protractor to the floor to mark the center. During the dance,

they use the strings to check the angle the dancers make with the center point and use the tick marks to make sure the two dancers are the same distance from the center. (A single unmarked string works fine, but is less convenient to use.)



**Dilation Dance:** Mark a 2 m piece of exercise elastic band at the  $\frac{1}{4}$ ,  $\frac{1}{2}$ , and  $\frac{3}{4}$  positions to check the ratio of distances. (Alternatively, you can use measuring tape or a meter stick, but these are less convenient to use.) Before the dance begins, choreographers tape the “Center” printout to the floor to mark the center. To check the dilation during the dance, one choreographer holds one end of the band at the “Center” mark, and the second choreographer uses the band to make sure the dancers are both in line with the center, stretching it to check whether the ratio of distances is correct. With just four marks on the band, you can check dilation scale factors such as  $\frac{1}{4}$ ,  $\frac{1}{3}$ ,  $\frac{1}{2}$ ,  $\frac{2}{3}$ ,  $\frac{3}{4}$ , and the corresponding inverse factors.

**Domain Markers:** Each of the dances will require the independent dancer to follow a restricted domain. To define the restricted domain, the choreographers put markers on the floor to show the path along which the independent dancer must dance. Sticky notes work well, as do colored poker chips. The choreographers should use enough markers to define a reasonably interesting path for the independent dancer. During the dance, the independent dancer should pause at each marker so that the choreographers can mark the position of the dependent dancer. Because the markers remain in place, they can be checked for accuracy either during or after the dance.

### Launch Part 1

Expect to spend about 10 minutes.

Pass out the worksheets (including the Function Dance Cards) to the class, and tell students that they will follow the directions on worksheet page 1 and that they will record their observations for each dance on their Dance Card.

Divide the class into teams of four or five students. Have each team choose an independent dancer and a dependent dancer, The remaining team members are choreographers, responsible for the accuracy of the dancing. Have students move their

desks to provide sufficient space for the dance performance. Alternatively, consider using a larger space if it is available.

Call up one team to demonstrate the process, and assign a translation dance to perform. Two of the team's choreographers tie a knot in the vector to set its length and then set it on the floor in direction of their choosing. Two choreographers tie a knot in a second rope to make it the same length as the vector. The independent dancer takes a position of his or her choice and the dependent dancer takes a position that matches the translation vector. The choreographers use the knotted rope to check the dependent dancer's location: they check the distance between dancers using the length of the rope and they check the angle by keeping the rope parallel to the vector. (If the floor is tiled, this task is even easier: the choreographers can just count floor tiles.) The independent dancer then moves to a different location, the dependent dancer matches the movement, and the choreographers again check the angle and distance. The dancers continue practicing with their choreographers' help to improve their ability to judge the proper location for their translation function.

Call students' attention to step 6 of the worksheet. The two dancers switch roles, and the choreographers create a restricted domain by placing sticky notes, poker chips or other marks on the floor. The independent dancer goes to the first mark, the dependent dancer moves accordingly, and the choreographers place a range mark. The independent dancer moves to the next domain mark, the dependent dancer moves simultaneously, and the choreographers place a second range mark.

Once the first two range marks are in place, call an end to the demonstration. Explain to the class that once the dance is finished and the range marks are in place, the choreographers will check the marks for accuracy. The dancers can repeat the dance several times to perfect their timing, and perhaps to add other coordinated motions. Remind students that the dependent dancer must match the motion of the independent dancer, and not the other way around.

Quickly review the tools the choreographers should use to check the positions of the dancers, and then have each team choose a dance, choreograph it, and practice it. Explain to students working on the rotation dance that they should perform the  $180^\circ$  dance before trying the more challenging  $90^\circ$  dance.

### ***Explore Part 1***

Expect to spend about 25 minutes.

Each dance team must choose their first dancers, decide what dance they will do, collect their props, and follow steps 1 through 9 on the worksheet for their first dance. Playing music in the classroom establishes an appropriate atmosphere in which students can plan and practice their dances.

Once each dance has been choreographed and performed, students should fill in the boxes on their Dance Cards for the dance. Encourage team members to discuss any questions about the relative speed, relative direction, or fixed points of the dance.

Circulate to observe the dancing and the choreography. You may need to give some additional hints to help the dependent dancer figure out where to be, or provide guidance to the choreographers in assessing the dancers' positions.

Once they've completed dance 1, each team should continue by choosing a different dance, and changing roles so that each team member has a chance to be a dancer as well as a choreographer. Continue circulating as students work, observing and questioning to make sure students understand how to check their dances for accuracy, and noting both insights and misconceptions to discuss during the class summary.

Have each team perform as many dances as time permits, but be sure to call a halt to the dancing early enough to conduct a summary discussion.

### ***Summarize Part 1***

Expect to spend about 10 minutes.

Gather the class and have students move the desks back into place. Ask students what they've learned, what was easy, what was hard, and what was interesting. As students describe their experiences and observations, you may want to ask questions such as these:

- *Which dances were easier, and which ones were more difficult? Why do you think this was?*
- *In which dances do the dancers always move at the same speed relative to each other?*
- *When the dancers move at different speeds, what determines the relative speed of the dependent dancer?*
- *In which dances do the dancers always move in the same direction relative to each other?*
- *When the dancers move in different directions, what determines the relative direction of the dependent dancer?*
- *In which dances are there fixed points (meeting points)?*
- *How many fixed points does each dance have? Where are these fixed points?*
- *For which dances are the restricted domain and corresponding range congruent, and for which are they similar?*

- *Which is harder, being an independent variable or being a dependent variable? How did it feel being a dependent variable?*

If the virtual function dance portion of the activity will be done on the following day, remind students that they must bring their worksheets and Dance Cards back the next day.

### ***Launch Part 2***

Expect to spend about 10 minutes.

In the virtual dances, students use the pages of **Dance the Dependent Variable.gsp** to perform the role of the dependent dancer for several different dances. With a block schedule, this part of the activity could be done on the same day as the Physical Function Dance; otherwise plan to do it the following day.

Tell students that they will be following the directions on page 2 of the worksheet. Call their attention to the Dance Cards, and tell them that they will record their observations for each dance on page 2 of their Dance Card.

Project page 1 of **Dance the Dependent Variable.gsp** and call two students to the computer. Tell one student to be the dependent dancer and operate the mouse, and tell the other student to be the choreographer/recorder. The choreographer reads worksheet step 12, and the dancer drags point  $p$  to position the circle over the independent variable  $a$ . The dancer presses the *Ready...Set...Go* button, waits for the countdown, and tries to follow point  $a$  as it moves around the restricted domain.

Tell the class that it's not easy to get the dance right the first time they try it, that they can practice several times, and that they can reduce the speed of the animation to make it easier. Students don't need to get each dance perfect—they should practice enough to get reasonably good, fill in the behavior information on their Dance Cards, and move on.

Depending on the class, you may want to ask several students to volunteer how they would fill in the various Dance Card boxes for page 1. Try to get multiple answers for each of the boxes, to emphasize both that different students will express their observations differently and that different descriptions can complement each other.

### ***Explore Part 2***

Expect students to spend about 25 minutes.

Send students to the computers in pairs. Emphasize that each member of the pair should practice each function dance at least once and that both members of the pair should agree on their description of the function behavior on their Dance Cards before moving on to the next page.



If iPads are available, students may want to do these virtual dances using Sketch Explorer for the iPad with the sketch **Dance the Dependent Variable iPad.gsp**.

Circulate as students work, making sure that they understand how to operate the sketch, asking questions about the mathematics they are experiencing, and noting discussion points for the class summary.

Students who finish early can go to page Choreograph to create their own function dance for their classmates to perform.

Save time for discussion at the end of the class period.

### ***Summarize Part 2***

Expect to spend about 10 minutes.

Solicit students' observations about the behaviors of different functions. Here are some specific questions to ask.

- *How did the behavior of some function families make their dances more difficult to perform (both physically and virtually) than others?*
- *Compare the two experiences—dancing with your feet vs. dancing with your fingers.*
- *How are they similar and how are they different?*
- *Which is easier, which is harder, and which is more accurate?*
- *How does each contribute to your understanding of functions?*

### ***Assess***

As students leave, ask them to fill out an exit ticket describing one interesting thing they learned, and one thing that they found challenging.

### ***Extensions***

Students can practice their physical function dances with music and then perform them for the entire class or for another class.

Students can make a video in which they demonstrate the function concepts they've learned through dance.

### ***Related Activities:***

- *ID the Suspects—Identify Functions*
- *Family Resemblances—Identify Function Families*

- *Reflection Challenges—The Reflection Family*
- *Rotation Challenges—The Rotation Family*
- *Dilation Challenges—The Dilation Family*
- *Translation Challenges—The Translation Family*
- *Family Relationships—Rotation, Translation, and Translation Families*

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Angle:  $90^\circ$

Angle:  $180^\circ$

Scale: 0.5

Scale: 2.0

Center

Mirror



