Notes for the Teacher

Students investigate numbers and their factors by manipulating arrays of counters. Students choose the number of counters for their array and then change the dimensions of the array while keeping the total number of counters the same. They pose and explore questions such as, “If I can represent 12 as a 3 × 4 array, can I also show it as a 4 × 3 array? What numbers form square arrays? How many ways can I arrange 23 counters so that every row of my array contains an equal number of counters? Do larger numbers have more factors than smaller factors?”

Objectives:

- Students will use a rectangular array model to find all factors of given numbers.
- Students will see the relationship between the pictorial representation of multiplication and its symbolic form.
- Students will make and record observations about relationships and patterns between numbers and their factors.

Common Core Mathematical Practices: (1) Make sense of problems and persevere in solving them; (2) Reason abstractly and quantitatively; (3) Construct viable arguments and critique the reasoning of others; (5) Use appropriate tools strategically; (7) Look for and make use of structure.

Common Core State Standards: 3.OA5, 6, 7, 9; 4.OA4, 5

Grade Range: Grades 3–4

Introduce:

Open **Boxed Counters--Find Factors with an Array Model.gsp** and distribute the worksheet. Use a projector to show the sketch page “Factor Array.” Ask, “How many red counters are shown?” (12) Invite a volunteer to drag the blue point using the **Arrow** tool while the other students watch. Ask students to describe what they see. Here are some possible observations:

- *Dragging the blue point changes the dimensions of the black rectangle.*
- *Dragging the blue point changes how many red counters are in each row.*
- *No matter how you drag the blue point, there are always 12 counters.*
- *If you drag the blue point far enough to the right, then all 12 counters appear in a single row. If you drag the blue point far enough to the left, all 12 counters appear in a single column.*

- *Sometimes there are the same number of counters in each row. For example, I can make 3 rows of 4 counters each or 6 rows of 2 counters each.*
- *When there are the same number of counters in each row, I can drag the black rectangle so that it fits exactly around the counters with none left over.*
- *When you make 4 rows of 3 counters, you can say that 4 × 3 = 12.*
- *When you make 4 rows of 3 counters, you can say that 4 and 3 are both factors of 12.*

Allow students to gradually develop the ideas described above, including the idea of factor. (If the word *factor* is new to students, help students make sense of it both visually, in relation to the width or height of the rectangle, and symbolically, in relation to the equivalent multiplication facr.) Continue by asking, “By dragging the blue point can you find all the different factors of 12?” Ask one or more volunteers to demonstrate by using the Sketchpad model.

A systematic approach might be to start with the blue point at a location where all 12 counters lie in a single row and then slowly drag the blue point to the left, looking for arrangements of the red counters where there are the same number of counters per row. To name these particular arrangements, establish the convention of identifying a rectangular array first by the number of rows and then by the number of counters in each row. For example, 2 rows of 6 counters is a 2 × 6 array, while 6 rows of 2 counters is a 6 × 2 array.

Students should find and list 6 rectangular arrays with 12 counters on their worksheet. These arrays are: 1 × 12, 12 × 1, 2 × 6, 6 × 2, 3 × 4, and 4 × 3. Based on the dimensions of these arrays, ask students to list the factors of 12. The factors are 1, 2, 3, 4, 6, and 12. Don’t worry if students have not found a systematic way to find and list all the factors. They will develop strategies as they explore on their own.

Demonstrate how to change the number of red counters shown on the sketch page. Use the Arrow tool to highlight 12 in the counters edit box and enter a new number using the computer keyboard. Then click anywhere outside the edit box.

Explain that for each number on their worksheet, students will use the array model to find its factors. Tell students to be on the lookout for any patterns or relationships they observe while working.

Explore:

Assign students to partners and send them in pairs to the computers. Have students open **Boxed Counters--Find Factors with an Array Model.gsp** and go to the page "Factor Arrays." Be sure students understand how to complete the table on their worksheet.

When students enter a large value like 36 into the *counters* edit box, their arrays may extend off the screen. Point to the scroll bars along the bottom and right side of the sketch page as a way to help them see what's happening off the visible portion of the screen.

As you circulate, observe students as they work. Have students found a systematic way to find all factors? Do students see a relationship between the dimensions of the rectangles and the factors of a number? Are students discovering that it is easy to build rectangular arrays for some numbers (those with many factors) but not others (primes)?

Discuss:

Call students together to discuss and summarize what they have learned. Review the factors students found for the numbers on their worksheets.

Answers:

Factors of Numbers

6: 1, 2, 3, 6
12: 1, 2, 3, 4, 6, 12
7: 1, 7
14: 1, 2, 7, 14
9: 1, 3, 9
18: 1, 2, 3, 6, 9, 18
36: 1, 2, 3, 4, 6, 9, 12, 18, 36
10: 1, 2, 5, 10
20: 1, 2, 4, 5, 10, 20
40: 1, 2, 4, 5, 8, 10, 20, 40
13: 1, 13
26: 1, 2, 13, 26
29: 1, 29

Then discuss any observations students made. Here are some possible student comments:

- *We thought that larger numbers would have more factors than smaller numbers, but 29 is the largest number in the table, and it only had two factors.*
- *Each number has 1 and itself as a factor. These were the arrays that were a single row or a single column.*

- *The numbers 7, 13, and 29 have only 1 and themselves as factors. We thought all odd numbers had only two factors, but then we saw that 9 has three factors.*
- *All the even numbers have 2 as a factor.*
- *If a number is a factor of another number, then its factors are also factors of the other number. For example, 6 is a factor of 12, and all of the factors of 6 are also factors of 12.*
- *For 9 and 36, we could make square arrays. That's because 9 and 36 are both square numbers.*
- *All the numbers had an even number of factors, except 9 and 36. They had an odd number of factors. That's because one of the arrays for 9 was 3 × 3 and one of the arrays for 36 was 6 × 6. Those were both square arrays. We didn't count the same number twice as a factor.*
- *For each non-square array, we found another array that used the same factors. For example, for 10, we found a 2 × 5 array and a 5 × 2 array. The factors are the same for both: 2 and 5.*

Project **Boxed Counters--Find Factors with an Array Model.gsp.** Discuss as many observations as time allows, using the sketch to illustrate as appropriate. Students may have questions they wish to explore further by finding other examples, such as "What other numbers have only 2 factors? What other numbers form square arrays?"

If your students are ready to explore the factors of numbers greater than 100, consider using the page "Larger Numbers." The counters on this page are smaller and more closely packed, allowing students to view more counters on the page.

Related Activities:

- *Dancing Factors—Find Factors of a Number*
- *Dynamic Number Grids—Multiples and Patterns*
- *Dynamic Number Grids—Clues You Can Use*
- *Factors in Blue and Gold—Explore Patterns of Factors*

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