Notes for the Teacher

Students are given a random value between 1 and 63 on one side of a scale and must balance it by selecting letters whose values are powers of 2 (1, 2, 4, 8, 16, and 32). Students write equations to represent their balance model and then make conjectures about the special properties of the powers of 2 based on their observations.

Objectives:

- Students will use a balance model to explore early algebraic concepts.
- Given a set of possible addends, students will select numbers whose sum equals a given target value.
- Students will develop strategies for finding combinations of addends that equal a target sum.
- Students will pose questions and make observations and conjectures about using the powers of 2 to make sums.

Common Core Mathematical Practices: (1) Make sense of problems and persevere in solving them; (2) Reason abstractly and quantitatively; (3) Construct viable arguments and critique the reasoning of others; (5) Use appropriate tools strategically; (7) Look for and make use of structure.

Common Core State Standards: 6.EE2, 6

Grade Range: Grades 4–7

Introduce:

Open **Balance--Balancing with Powers of Two.gsp** and distribute the worksheet. Use a projector to show sketch page "Balance." Point out that in the current problem, n has a value of 30. Explain that each letter on the left, a–f, has a value as well. Tell students that their goal is to find combinations of the letters a–f whose sum is equal to n. Each letter can only be used once. Ask, "What letters can we drag onto the left side of the scale so that they balance with n?"

Have volunteers suggest letters that might balance the scale. Show students how to use the **Arrow** tool to drag the letters onto the balance scale to check. When the scale balances, ask students to represent the model by writing an equation using the letters. For $n = 30$, this equation can be written as:

$$b + c + d + e = n$$

Equivalently, students might put *n* on the left side of the equation or write the letter *b*, *c*, *d*, and *e* in a different order. Explore these possibilities by writing several equivalent equations. Then substitute the values to show the equations are true statements.

Press the *New Problem* button with the **Arrow** tool to get a new value for *n*. Explain that the values for *n* are random numbers between 1 and 63. Have volunteers come to the computer and work through additional problems in front of the class until students are clear about their task.

Tell students that they will solve several problems like these with a partner. Ask students to pay attention to anything interesting that they notice while solving the problems and to write down any questions and conjectures they have. Questions should naturally arise during the course of the activity.

Explore:

Assign students to partners and send them in pairs to the computers. Have students open **Balance--Balancing with Powers of Two.gsp** and go to page "Balance." Be sure students understand how to record their solutions by writing the value for *n* and the matching equation using only letters.

As you circulate, observe students as they work. What questions are they asking? What observations are they making? If students are struggling to make conjectures, help them by asking questions such as the following:

- *Is there ever more than one combination of letters that will add up to n?*
- *Is there a value of n you cannot make with the letters a–f?*
- *What do you notice about the values of the letters a–f?*

Discuss:

Call students together to discuss and summarize what they have learned. Have volunteers share the different sums they found. Then ask students to discuss the different observations and conjectures they made. Here are some sample student responses:

- *For every value of n, we found only one group of numbers whose sum was n. You can rearrange the order of the letters to create a different equation, but we counted those equations, such as b + c = n and c + b = n, as being the same.*
- *For some values of n like 4 and 16, we could only write c = n or e = n.*
- *We noticed that for all the odd values of n, we always used the letter a, which is 1, in our sums.*

- *We found a way to make each value of n that the computer gave us. We guessed that you could make any sum from 1 to 63 using just the letters a–f.*
- *It's kind of amazing that you can make any sum from 1 to 63 with just six letters!*
- *We noticed that a + b = 3, which is 1 less than c, and a + b + c = 7, which is 1 less than d, and so on. We thought that was cool.*
- *When we were working, we always started with the letter whose value was closest to n. For example, to make a sum of 18, we started with e. It made it easier to see what we needed to add on to balance n.*
- *When n = 63, we need to use all the letters a–f to balance the scale.*
- *We noticed that a × 2 = b, b × 2 = c, c × 2 = d, and so on.*
- *We figured that if you wanted to make sums larger than 63, you'd have to include a new letter, g, with a value of 64 since f × 2 = 64. Since 64 + 32 + 16 + 8 + 4 + 2 + 1 = 127, the largest sum you could make would be 127. That means you could write any number from 1 to 127 as a sum using the letters a-g at most once.*

Mathematical Note for Your Information:

Let's examine how the numbers 15 and 47 are represented using the powers of two available in the sketch:

$15 = \mathbf{1} \times 8 + \mathbf{1} \times 4 + \mathbf{1} \times 2 + \mathbf{1} \times 1$

$47 = \mathbf{1} \times 32 + \mathbf{0} \times 16 + \mathbf{1} \times 8 + \mathbf{1} \times 4 + \mathbf{1} \times 2 + \mathbf{1} \times 1$

Written another way, we could say 15 = 1111 (base 2) and 47 = 101111 (base 2).

When students use the numbers 1, 2, 4, 8, 16, and 32 to form a sum between 1 to 63, they are essentially finding its equivalent base 2 (binary) representation. That is why it is possible to form every sum from 1 to 63 using this limited set of numbers and why each sum can only be formed in just one way.

Related Activities:

- *Arranging Addends—Target Sum Puzzles*
- *Balance—Introducing Symbols and Equality*
- *Balance—Reasoning with Inequalities*

- *Balance—Solving for Unknowns, Part One*
- *Balance—Solving for Unknowns, Part Two*
- *Balance—Solving for Unknowns, Part Three*

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